

| UČNI NAČRT PREDMETA / COURSE SYLLABUS  |                           |   |                              |  |   |             |
|--|---------------------------|---|------------------------------|--|---|-------------|
| <b>Predmet:</b>  |                           | Uvod v diferencialno geometrijo                 |                              |  |   |             |
| <b>Course title:</b>   |                           | Introduction to differential geometry           |                              |  |   |             |
| <b>Študijski program in stopnja</b><br>Study programme and level             |                           | <b>Študijska smer</b><br>Study field            |                              | <b>Letnik</b><br>Academic year                 | <b>Semester</b><br>Semester             |             |
| Univerzitetni študijski program<br>Matematika                                |                           | ni smeri  |                              | 3  | prvi                                    |             |
| First cycle academic study<br>programme Mathematics                          |                           | none  |                              | 3  | first                                   |             |
| <b>Vrsta predmeta / Course type</b>  |                           |   |                              | izbirni  |   |             |
| <b>Univerzitetna koda predmeta / University course code:</b>                 |                           |   |                              | M0230  |   |             |
| <b>Predavanja</b><br>Lectures  | <b>Seminar</b><br>Seminar | <b>Vaje</b><br>Tutorial                         | <b>Klinične vaje</b><br>work | <b>Druge oblike</b><br>študija                 | <b>Samost. delo</b><br>Individ.<br>work | <b>ECTS</b> |
| 30   |                           | 30  |                              |  | 90                                      | 5           |
| <b>Nosilec predmeta / Lecturer:</b>  |                           | prof. Pavle Saksida, prof. Sašo Strle           |                              |  |   |             |
| <b>Jeziki / Languages:</b>   |                           | <b>Predavanja / Lectures:</b> slovenski/Slovene |                              |  |   |             |
|  |                           | <b>Vaje / Tutorial:</b> slovenski/Slovene       |                              |  |   |             |
| <b>Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:</b> |                           |   |                              | <b>Prerequisites:</b>                          |   |             |
| Opravljen predmeta Analiza 2a in Analiza 2b.                                 |                           |   |                              | Completed courses Analysis 2a and Analysis 2b. |   |             |
| <b>Vsebina:</b>  |                           |   |                              | <b>Content (Syllabus outline):</b>             |   |             |

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| <p>Uvod: Definicija krivulj in ploskev in primeri. Abstraktne krivulje in ploskve ter krivulje in ploskve, vložene v tridimenzionalni realni prostor. Spremljajoči trieder krivulje. Freneteve formule. Karte, atlas, orientabilnost, ponovitev pojmov Eulerjeve karakteristike in rodu ploskve. Gladke ploskve.</p> <p>Metrika: Izometrija ploskev. Pojem metrike in izometričnosti.</p> <p>Ukrivljenost: Druga fundamentalna forma ploskve v prostoru. Normalna in geodetska ukrivljenost krivulje na ploskvi. Gaussova ukrivljenost ploskve. Različni opisi in interpretacije ukrivljenosti. Povprečna ukrivljenost in minimalne ploskve. Geodetske krivulje. Geodetski koordinatni sistem.</p> <p>Gaussov Theorema egregium: Gaussova ukrivljenost kot izometrična invariants. Klasifikacija ploskev s konstantno ukrivljenostjo.</p> <p>Gauss-Bonnetev izrek: Lokalna in globalna različica tega izreka. Povezava med Gaussovo ukrivljenostjo kot geometrijsko in Eulerjevo karakteristiko kot topološko invarianto. Izrek o številu stacionarnih točk vektorskega polja na sklenjeni orientabilni ploskvi. Izrek o alternirajoči vsoti minimov, maksimov in sedel funkcije na sklenjeni orientabilni ploskvi.</p> | <p>Introduction: Definition of a curve and of a surface with some examples. Abstract curves and surfaces. Curves and surfaces, embedded in three-dimensional real space. Frenet frame. Frenet formulae. Charts, atlases, orientability, revision of the Euler characteristic and genus. Smooth surfaces.</p> <p>Metric: Isometry of surfaces. The concept of the metric. Isometry.</p> <p>Curvature: Second fundamental form of an imbedded surface. Normal and geodesic curvatures of a curve on a surface. Gaussian curvature. Various descriptions and interpretations of the Gaussian curvature. Mean curvature and minimal surfaces. Geodesic curves. Geodesic coordinate system.</p> <p>Gauss' Theorema Egregium: Gaussian curvature as an isometric invariant. Classification of surfaces with constant curvature.</p> <p>Gauss-Bonnet Theorem: The local and the global versions of this theorem. Relation between the Euler characteristic (a topological invariant) and the Gaussian curvature (geometric invariant). Algebraic number of stationary points of a vector field on a closed surface. The alternating sum of minima, saddles and maxima of a smooth function on a surface.</p> |
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### Temeljni literatura in viri / Readings:

A. Pressley, Elementary Differential Geometry, Springer, Berlin, 2010

Vidav: Diferencialna geometrija, DMFA-založništvo, Ljubljana, 1989.

M. P. do Carmo: Differential Geometry of Curves and Surfaces, Prentice Hall, Englewood Cliffs, 1976.

D. W. Henderson: Differential Geometry : A Geometric Introduction, Prentice Hall, Upper Saddle River, 1997.

I. M. Singer, J. A. Thorpe: Lecture Notes on Elementary Topology and Geometry, Springer, New

York-Heidelberg, 1976.

#### **Cilji in kompetence:**

Študent spozna osnovne pojme diferencialne geometrije: metriko, ukrivljenost in geodetske krivulje. Ti pojmi so konstruirani in obravnavani na ploskvah, ki so najenostavnejše netrivialno okolje za obravnavo diferencialno-geometričnih vsebin. Pojmi so predstavljeni v glavnem na ekstrinzični način, študent pa se vseeno spozna z razliko med ekstrinzičnim in intrinzičnim pogledom na geometrijo. Prek Gauss-Bonetevega izreka je poudarjena povezava geometrije s topologijo.

#### **Objectives and competences:**

Students get acquainted with the elementary notions of differential geometry, namely the metric, the curvature and the geodesic curves. These notions are treated on surfaces which are the simplest nontrivial context for the study of differential geometric constructions. The constructions presented are mainly extrinsic, but nevertheless, the students get acquainted with the difference between the extrinsic and the intrinsic. The relationship between topology and geometry is established by means of the Gauss-Bonnet theorem.

#### **Predvideni študijski rezultati:**

Znanje in razumevanje: Razumevanje osnovnih diferencialno geometrijskih pojmov: metrike, ukrivljenosti in geodetske krivulje. Razumevanje razlike med geometrijskimi in topološkimi lastnostmi prostorov. Sposobnost računske obravnave navedenih pojmov. Uporaba: Opis nekaterih uporab diferencialne geometrije v mehaniki in v drugih vejah fizike. Uporaba geometrije pri topološki obravnavi ploskev. Uporaba geometrije pri reševanju nekaterih diferencialnih enačb.

Refleksija: Razumevanje teorije na podlagi uporabe.

Prenosljive spretnosti – niso vezane le na en predmet: Za razumevanje predmeta je potrebno solidno obvladanje nekaterih vsebin iz Analize 1 in Analize 2a in 2b ter iz Linearne algebre. Študent se nauči uporabljati znanje, pridobljeno pri drugih predmetih.

#### **Intended learning outcomes:**

Knowledge and understanding: Understanding of the fundamental notions of differential geometry: the metric, the curvature and the geodesic curves. Understanding of the difference between topological and geometric properties of spaces. Ability of computational treatment of the notions mentioned above. Application: Some applications of differential geometry in mechanics and other branches of physics. Use of geometry in the topological study of surfaces. Use of differential geometry in solving certain differential equations.

Reflection: Understanding of the theory from the applications.

Transferable skills: Proficiency in certain chapters of Analysis 1, Analysis 2a and 2b and Linear algebra is needed for successful study of differential geometry. Students learn how to use previously acquired knowledge in the study

Spretnost uporabe domače in tuje literature.

of new subjects.

Skills related to the use of literature in different languages.

**Metode poučevanja in učenja:**

Predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**

Lectures, exercises, homework, consultations

Delež (v %) /

Weight (in %)

**Načini ocenjevanja:**

**Assessment:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):  
2 kolokvija namesto izpita iz vaj, izpit iz vaj,

izpit iz teorije

ocene: 1-5 (negativno), 6-10 (pozitivno)  
(po Statutu UL)

50%  
50%

Type (examination, oral, coursework, project):

2 midterm exams instead of written exam, written exam

oral exam

grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

Pavle Saksida:

– SAKSIDA, Pavle. Nahm's equations and generalizations Neumann system. Proceedings of the London Mathematical Society, ISSN 0024-6115, 1999, let. 78, št. 3, str. 701-720 [COBISS.SI-ID 8853849]

– SAKSIDA, Pavle. Neumann system, spherical pendulum and magnetic fields. Journal of physics. A, Mathematical and general, ISSN 0305-4470, 2002, vol. 35, no. 25, str. 5237-5253 [COBISS.SI-ID 11920217]

– SAKSIDA, Pavle. On zero-curvature condition and Fourier analysis. Journal of physics. A, Mathematical and theoretical, ISSN 1751-8113, 2011, vol. 44, no. 8, 085203 (19 str.) [COBISS.SI-ID

15909465]

Sašo Strle:

– STEFANOVSKA, Aneta, STRLE, Sašo, KROŠELJ, Peter. On the overestimation of the correlation dimension. *Physics letters. Section A*, ISSN 0375-9601. [Print ed.], 1997, vol. 235, str. 24-30 [COBISS.SI-ID 607828]

– STRLE, Sašo. Bounds on genus and geometric intersections from cylindrical end moduli spaces. *Journal of differential geometry*, ISSN 0022-040X, 2003, vol. 65, no. 3, str. 469-511 [COBISS.SI-ID 13135193]

– OWENS, Brendan, STRLE, Sašo. Rational homology spheres and the four-ball genus of knots. *Advances in mathematics*, ISSN 0001-8708, 2006, vol. 200, iss. 1, str. 196-216 [COBISS.SI-ID 13875033]