

UČNI NAČRT PREDMETA / COURSE SYLLABUS						
<b>Predmet:</b>		Teorija mere				
<b>Course title:</b>		Measure theory				
<b>Študijski program in stopnja</b> Study programme and level		<b>Študijska smer</b> Study field		<b>Letnik</b> Academic year	<b>Semester</b> Semester	
Magistrski študijski program Finančna matematika		ni smeri		1 ali 2	prvi ali drugi	
Master's study programme Financial Mathematics		none		1 or 2	first or second	
<b>Vrsta predmeta / Course type</b>				izbirni		
<b>Univerzitetna koda predmeta / University course code:</b>				M2110		
<b>Predavanja</b> Lectures	<b>Seminar</b> Seminar	<b>Vaje</b> Tutorial	<b>Klinične vaje</b> work	<b>Druge oblike</b> študija	<b>Samost. delo</b> Individ. work	<b>ECTS</b>
45		30			105	6
<b>Nosilec predmeta / Lecturer:</b>		doc. Marko Kandić, prof. Bojan Peter Magajna, prof. Roman Drnovšek				
<b>Jeziki / Languages:</b>		<b>Predavanja / Lectures:</b> slovenski/Slovene, angleški/English				
		<b>Vaje / Tutorial:</b> slovenski/Slovene, angleški/English				
<b>Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:</b>				<b>Prerequisites:</b>		
<b>Vsebina:</b>				<b>Content (Syllabus outline):</b>		

<p>Mere: <math>\sigma</math>-algebre, pozitivne mere, zunanje mere, Caratheodoryjev izrek, razširitev mere iz algebre na sigma algebro, Borelove mere na <math>\mathbb{R}</math>, Lebesguova mera na <math>\mathbb{R}</math>.</p> <p>Merljive funkcije: aproksimacija s stopničastimi funkcijami, načini konvergence funkcijskih zaporedij, izrek Jegorova.</p> <p>Integracija: integral nenegativne funkcije, izrek o monotoni konvergenci, Fatoujeva lema, integral kompleksne funkcije, izrek o dominirani konvergenci, primerjava Riemannovega in Lebesguovega integrala, izrek Jegorova.</p> <p>Produktne mere: konstrukcija produktnih mer, monotoni razredi, Tonellijev in Fubinijev izrek, Lebesguov integral na <math>\mathbb{R}^n</math>.</p> <p>Kompleksne mere: predznačene mere, Hahnov in Jordanov razcep, kompleksne mere, variacija mere, absolutna zveznost in vzajemna singularnost, Lebesgue-Radon-Nikodymov izrek.</p> <p><math>L^p</math>-prostori: neenakosti Jensena, Hölderja in Minkovskega, omejeni linearni funkcionali, dualni prostori.</p> <p>Integriranje na lokalno kompaktnih prostorih: pozitivni linearni funkcionali na <math>C_c(X)</math>, Radonove mere, Rieszov izrek, Lusinov izrek, gostota prostora <math>C_c(X)</math> v <math>L^p</math>-prostorih.</p> <p>Odvajanje mer na <math>\mathbb{R}^n</math> : odvajanje mer, absolutno zvezne in funkcije z omejeno totalno variacijo.</p>	<p>Measures: <math>\sigma</math>-algebras, positive measures, outer measures, Caratheodory's theorem, extension of measures from algebras to <math>\sigma</math>-algebras, Borel measures on <math>\mathbb{R}</math>, Lebesgue measure on <math>\mathbb{R}</math>.</p> <p>Measurable functions: approximation by step functions, modes of convergence of sequences of functions, Egoroff's theorem.</p> <p>Integration: integration of nonnegative functions, Lebesgue monotone convergence theorem, Fatou's lemma, integration of complex functions, Lebesgue dominated convergence theorem, comparison with Riemann's integral.</p> <p>Product measures: construction of product measures, monotone classes, Tonelli's and Fubini's theorem, the Lebesgue integral on <math>\mathbb{R}^n</math>.</p> <p>Complex measures: signed measures, the Hahn and the Jordan decomposition, complex measures, variation of a measure, absolute continuity and mutual singularity, the Lebesgue-Radon-Nikodym theorem.</p> <p><math>L^p</math>-spaces: inequalities of Jensen, Hölder and Minkovski, bounded linear functionals, dual spaces.</p> <p>Integration on locally compact spaces: positive linear functionals on <math>C_c(X)</math>, Radon measures, Riesz representation theorem, Lusin's theorem, density of <math>C_c(X)</math> in <math>L^p</math>-spaces.</p> <p>Differentiation of measures on <math>\mathbb{R}^n</math> : differentiation of measures, absolutely continuous and functions of bounded variation.</p>
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### Temeljni literatura in viri / Readings:

C. D. Aliprantis, O. Burkinshaw: Principles of Real Analysis, 3rd edition, Academic Press, San Diego, 1998.

R. Drnovšek: Rešene naloge iz teorije mere, DMFA-založništvo, Ljubljana, 2001.

G. B. Folland: Real Analysis : Modern Techniques and Their Applications, 2nd edition, John Wiley & Sons, New York, 1999.

M. Hladnik: Naloge in primeri iz funkcionalne analize in teorije mere, DMFA-založništvo, Ljubljana, 1985.

S. Kantorovitz: Introduction to Modern Analysis, Oxford Univ. Press, 2003.

B. Magajna: Osnove teorije mere, DMFA-založništvo, Ljubljana, 2011.

W. Rudin: Real and Complex Analysis, 3rd edition, McGraw-Hill, New York, 1987.

### Cilji in kompetence:

Študent pridobi znanje osnov teorije mere, ki jih potrebuje za razumevanje osnov sodobnega verjetnostnega računa, statistike in funkcionalne analize.

### Objectives and competences:

Students acquire basic knowledge of measure theory needed to understand probability theory, statistics and functional analysis.

### Predvideni študijski rezultati:

Znanje in razumevanje: Razumevanje osnovnih pojmov teorije mere.

Uporaba: Teorija mere sodi med temeljne predmete na 2. stopnji študija matematike, saj je nujno potrebna za razumevanje osnov sodobnega verjetnostnega računa, statistike in funkcionalne analize. Poleg tega njena uporaba sega tudi v naravoslovje in druga področja znanosti kot na primer ekonomijo.

### Intended learning outcomes:

Knowledge and understanding: understanding basic concepts of measure and integration theory.

Application: measure theory is a part of the basic curriculum of the graduate study of mathematics since it is needed in other areas, for example, in probability calculus, statistics and functional analysis. It is useful also in other sciences, for example in economy.

<p>Refleksija: Razumevanje teorije na podlagi primerov uporabe.</p> <p>Prenosljive spretnosti – niso vezane le na en predmet: Sposobnost abstraktnega razmišljanja. Spretnost uporabe domače in tuje literature.</p>	<p>Reflection: understanding of the theory on the basis of examples of application.</p> <p>Transferable skills: Ability to use abstract methods to solve problems. Ability to use a wide range of references and critical thinking.</p>

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije
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**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations
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Delež (v %) /

Weight (in %)

**Načini ocenjevanja:**

**Assessment:**

<p>Način (pisni izpit, ustno izpraševanje, naloge, projekt):</p> <p>izpit iz vaj (2 kolokvija ali pisni izpit)</p> <p>ustni izpit</p> <p>Ocene: 5 (negativno), 6-10 (pozitivno) (po Statutu UL)</p>	<p>50%</p> <p>50%</p>	<p>Type (examination, oral, coursework, project):</p> <p>2 midterm exams instead of written exam, written exam</p> <p>oral exam</p> <p>Grading: 5 (fail), 6-10 (pass) (according to the Statute of UL)</p>
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**Reference nosilca / Lecturer's references:**

<p>Roman Drnovšek:</p> <p>– DRNOVŠEK, Roman. On invariant subspaces of Volterra-type operators. Integral equations and operator theory, ISSN 0378-620X, 1997, let. 27, št. 1, str. 1-9 [COBISS.SI-ID 7038553]</p> <p>– DRNOVŠEK, Roman. Spectral inequalities for compact integral operators on Banach function spaces. Mathematical proceedings of the Cambridge Philosophical Society, ISSN 0305-0041, 1992,</p>
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let. 112, str. 589-598 [COBISS.SI-ID 8169561]

– DRNOVŠEK, Roman. A generalization of Levinger's theorem to positive kernel operators. Glasgow mathematical journal, ISSN 0017-0895, 2003, vol. 45, part 3, str. 545-555 [COBISS.SI-ID 12825945]

Marko Kandić:

– DRNOVŠEK, Roman, KANDIĆ, Marko. More on positive commutators, Journal of mathematical analysis and applications. ISSN 0022-247X. - Vol. 373, iss. 2 (2011), str. 580-584. [COBISS.SI-ID 15672409]

– KANDIĆ, Marko. Ideal-triangularizability of nil-algebras generated by positive

operators, Proceedings of the American Mathematical Society. ISSN 0002-9939. - Vol. 139, no. 2 (2011), str. 485-490 [COBISS.SI-ID 15710809]

– KANDIĆ, Marko. Sets of matrices with singleton spectra generated by positive matrices, Linear Algebra and its Applications. ISSN 0024-3795. - Vol. 496 (2016), str. 463-474. [COBISS.SI-ID 17602137]

Bojan Peter Magajna:

– MAGAJNA, Bojan. Uniform approximation by elementary operators. Proceedings of the Edinburgh Mathematical Society, ISSN 0013-0915, 2009, vol. 52, part 3, str. 731-749 [COBISS.SI-ID 15352921]

– MAGAJNA, Bojan. Fixed points of normal completely positive maps on  $B(H)$ . Journal of mathematical analysis and applications, ISSN 0022-247X. [Print ed.], 2012, vol. 389, iss. 2, str. 1291-1302 [COBISS.SI-ID 16227673]

– MAGAJNA, Bojan. Sums of products of positive operators and spectra of Lüders operators. Proceedings of the American Mathematical Society, ISSN 0002-9939, 2013, vol. 141, no. 4, str. 1349-1360 [COBISS.SI-ID 16603481]