

| UČNI NAČRT PREDMETA / COURSE SYLLABUS  |                |   |                      |                                    |                     |                 |  |
|--|----------------|---|----------------------|------------------------------------|---------------------|-----------------|--|
| <b>Predmet:</b>  |                | Numerična aproksimacija in interpolacija                          |                      |                                    |                     |                 |  |
| <b>Course title:</b>   |                | Numerical approximation and interpolation                         |                      |                                    |                     |                 |  |
| <b>Študijski program in stopnja</b>  |                | <b>Študijska smer</b>   |                      | <b>Letnik</b>                      |                     | <b>Semester</b> |  |
| Study programme and level  |                | Study field   |                      | Academic year                      |                     | Semester        |  |
| Magistrski študijski program<br>Finančna matematika                          |                | ni smeri  |                      | 1 ali 2                            |                     | prvi ali drugi  |  |
| Master's study programme<br>Financial Mathematics                            |                | none  |                      | 1 or 2                             |                     | first or second |  |
| <b>Vrsta predmeta / Course type</b>  |                |   |                      | temeljni                           |                     |                 |  |
| <b>Univerzitetna koda predmeta / University course code:</b>                 |                |   |                      | M2406                              |                     |                 |  |
| <b>Predavanja</b>  | <b>Seminar</b> | <b>Vaje</b>   | <b>Klinične vaje</b> | <b>Druge oblike</b>                | <b>Samost. delo</b> | <b>ECTS</b>     |  |
| Lectures   | Seminar        | Tutorial  | work                 | študija                            | Individ. work       |                 |  |
| 45   |                | 30  |                      |                                    | 105                 | 6               |  |
| <b>Nosilec predmeta / Lecturer:</b>  |                | prof. Emil Žagar, prof. Marjetka Knez                             |                      |                                    |                     |                 |  |
| <b>Jeziki / Languages:</b>   |                | <b>Predavanja / Lectures:</b> slovenski/Slovene, angleški/English |                      |                                    |                     |                 |  |
|  |                | <b>Vaje / Tutorial:</b> slovenski/Slovene, angleški/English       |                      |                                    |                     |                 |  |
| <b>Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:</b> |                |   |                      | <b>Prerequisites:</b>              |                     |                 |  |
|  |                |   |                      |                                    |                     |                 |  |
| <b>Vsebina:</b>  |                |   |                      | <b>Content (Syllabus outline):</b> |                     |                 |  |
|  |                |   |                      |                                    |                     |                 |  |

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| <p>Aproksimacija funkcij: Izbira prostorov aproksimativnih funkcij. Polinomi. Trigonometrijski polinomi. Odsekoma polinomske funkcije. Stabilnost baz. Weierstrassov izrek. Pozitivni operatorji. Optimalni aproksimativni problem. Eksistenca in enoličnost elementa najboljše aproksimacije. Enakomerna konveksnost, stroga normiranost.</p> <p>Enakomerna aproksimacija s polinomi: Enoličnost za diskretni in zvezni primer. Alternacija residuala. Konstrukcija. Prvi in drugi Remesov postopek. Konvergenca. Polinomi Čebiševa. Posplošitve: Čebiševi sistemi funkcij, generalizirani polinomi.</p> <p>Metoda najmanjših kvadratov v zveznem in diskretnem primeru: Ortogonalni polinomi. Tričlenska rekurzivna formula. Gram-Schmidtova ortogonalizacija in numerično stabilnejše izvedbe. Reortogonalizacija. Navezava diskretnega in zveznega primera. Enakomerna konvergenca L2-aproksimacij.</p> <p>Interpolacija: Interpolacija s polinomi. Lagrangeva oblika interpolacijskega polinoma in ostanek. Baricentrična Lagrangeova interpolacija. Deljene diference. Newtonova oblika interpolacijskega polinoma, posplošena Hornerjeva shema. Divergenca interpolacijskih polinomov.</p> <p>Odsekoma polinomske funkcije, zlepki: Eulerjevi poligoni, interpolacija in aproksimacija v drugi normi. Kubični zlepki. B-zlepki kot baza prostora odsekoma polinomskih funkcij. Bézierove krivulje. Zlepki v dveh dimenzijah.</p> | <p>Approximation of functions: Spaces of approximation functions. Polynomials. Trigonometric polynomials. Piecewise polynomial functions. Stability of bases. Weierstrass' Theorem. Positive operators.</p> <p>Optimal approximation. Existence and uniqueness of the best approximation. Uniform convexity and strong normed spaces.</p> <p>Uniform approximation by polynomials:</p> <p>Uniqueness in the discrete and continuous case. Iteration of residuals. Construction. The first and the second Remes algorithm. Convergence. Chebyshev polynomials. Generalizations: Chebyshev systems, generalized polynomials.</p> <p>Continuous and discrete least squares:</p> <p>Orthogonal polynomials. Three-term recurrence. Gram-Schmidt orthogonalization, basic and stable version. Reorthogonalization.</p> <p>Connection between discrete and continuous case. Uniform convergence of L2-approximants.</p> <p>Interpolation: Polynomial interpolation. Lagrange form. Barycentric Lagrange interpolation. Divided differences. Newton form and generalized Horner scheme. Divergence of interpolating polynomials.</p> <p>Piecewise polynomial functions, splines: Euler polygons, interpolation and approximation in the second norm. Cubic splines. B-spline bases of piecewise polynomial functions. Bézier curves. Splines in two dimensions.</p> |
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**Temeljni literatura in viri / Readings:**

J. Kozak: Numerična analiza, DMFA-založništvo, Ljubljana, 2008.

R. L. Burden, J. D. Faires: Numerical Analysis, 8th edition, Brooks/Cole, Pacific Grove, 2005.

E. K. Blum: Numerical Analysis and Computation : Theory and Practice, Addison-Wesley, Reading, 1998.

Z. Bohte: Numerične metode, DMFA-založništvo, Ljubljana, 1991.

S. D. Conte, C. de Boor: Elementary Numerical Analysis : An Algorithmic Approach, 3rd edition, McGraw-Hill, Auckland, 1986.

C. de Boor: A Practical Guide to Splines, Springer, New York, 2001.

E. Isaacson, H. B. Keller: Analysis of Numerical Methods, John Wiley & Sons, New York-London-Sydney, 1994.

D. R. Kincaid, E. W. Cheney: Numerical Analysis : Mathematics of Scientific Computing, 3rd edition, Brooks/Cole, Pacific Grove, 2002.

#### **Cilji in kompetence:**

Slušatelj dopolni poznavanje analitičnih metod aproksimacije in interpolacije z numeričnimi. Ob domačih nalogah pridobljeno znanje praktično utrdi.

#### **Objectives and competences:**

Student supplements knowledge of analytical methods in approximation and interpolation by numerical aspects. By solving homeworks the obtained theoretical knowledge is consolidated.

#### **Predvideni študijski rezultati:**

Znanje in razumevanje: Razumevanje pojmov interpolacije in aproksimacije. Praktično obvladavanje numeričnih postopkov za konstrukcijo interpolacijskih oziroma aproksimacijskih funkcij.

Uporaba: Numerična konstrukcija interpolacijskih ali aproksimacijskih funkcij s pomočjo računalnika in ocenjevanje napak na podlagi teorije. Interpolacija in aproksimacija se uporabljata na mnogih področjih, še posebej pri računalniško podprtem grafičnem modeliranju.

Refleksija: Razumevanje teorije na podlagi uporabe.

#### **Intended learning outcomes:**

Knowledge and understanding: Understanding of interpolation and approximation. Ability of numerical algorithms for construction of interpolating or approximating functions.  
Application: Numerical construction of interpolating and approximating functions using a computer and error estimation based on theory. Interpolation and approximation are used in several fields, in particular in computer aided graphical modelling.

Reflection: Understanding of theory based through applications.

Transferable skills: Skill of using computer for solving numerical problems. Understanding

Prenosljive spretnosti – niso vezane le na en predmet: Spretnost uporabe računalnika pri reševanju matematičnih problemov. Razumevne razlik med eksaktnim in numeričnim računanjem.

differences between exact and numerical computing.

**Metode poučevanja in učenja:**

Predavanja, vaje, domače naloge, konzultacije.

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations.

**Načini ocenjevanja:**

Delež (v %) /

Weight (in %)

**Assessment:**

Način (domače naloge, pisni izpit, ustno izpraševanje, naloge, projekt):  
domače naloge ali project

pisni izpit

ustni izpit

Ocene: 1-5 (negativno), 6-10 (pozitivno)  
(po Statutu UL)

20%

40%

40%

Type (homeworks, examination, oral, coursework, project):  
homeworks or project

written exam

oral exam

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

Marjetka Knez:

– KRAJNC, Marjetka. Geometric Hermite interpolation by cubic  $G^1$  splines. Nonlinear Analysis, Theory, Methods and Applications, ISSN 0362-546X. [Print ed.], 2009, vol. 70, iss. 7, str. 2614-2626 [COBISS.SI-ID 15508569]

– KRAJNC, Marjetka. Interpolation scheme for planar cubic  $G^2$  spline curves. Acta applicandae mathematicae, ISSN 0167-8019, 2011, vol. 113, no. 2, str. 129-143 [COBISS.SI-ID 16215385]

– JAKLIČ, Gašper, KOZAK, Jernej, KRAJNC, Marjetka, VITRIH, Vito, ŽAGAR, Emil. High order

parametric polynomial approximation of conic sections. Constructive approximation, ISSN 0176-4276, 2013, vol. 38, iss. 1, str. 1-18 [COBISS.SI-ID 16716121]

Emil Žagar:

– ŽAGAR, Emil. On  $G^2$  continuous spline interpolation of curves in  $R^d$ . BIT, ISSN 0006-3835, 2002, vol. 42, no. 3, str. 670-688 [COBISS.SI-ID 12027993]

– KOZAK, Jernej, ŽAGAR, Emil. On geometric interpolation by polynomial curves. SIAM journal on numerical analysis, ISSN 0036-1429, 2004, vol. 42, no. 3, str. 953-967 [COBISS.SI-ID 13398617]

– JAKLIČ, Gašper, KOZAK, Jernej, VITRIH, Vito, ŽAGAR, Emil. Lagrange geometric interpolation by rational spatial cubic Bézier curves. Computer Aided Geometric Design, ISSN 0167-8396, 2012, vol. 29, iss. 3-4, str. 175-188 [COBISS.SI-ID 16207449]