

UČNI NAČRT PREDMETA / COURSE SYLLABUS							
<b>Predmet:</b>		Riemannove ploskve					
<b>Course title:</b>		Riemann surfaces					
<b>Študijski program in stopnja</b>		<b>Študijska smer</b>		<b>Letnik</b>		<b>Semester</b>	
<b>Study programme and level</b>		<b>Study field</b>		<b>Academic year</b>		<b>Semester</b>	
Magistrski študijski program Finančna matematika		ni smeri		1 ali 2		prvi ali drugi	
Master's study programme Financial Mathematics		none		1 or 2		first or second	
<b>Vrsta predmeta / Course type</b>				izbirni			
<b>Univerzitetna koda predmeta / University course code:</b>				M2314			
<b>Predavanja</b>	<b>Seminar</b>	<b>Vaje</b>	<b>Klinične vaje</b>	<b>Druge oblike študija</b>	<b>Samost. delo</b>		<b>ECTS</b>
<b>Lectures</b>	<b>Seminar</b>	<b>Tutorial</b>	<b>work</b>		<b>Individ. work</b>		
<b>30</b>	<b>15</b>	<b>30</b>			<b>105</b>		<b>6</b>
<b>Nosilec predmeta / Lecturer:</b>		prof. Barbara Drinovec Drnovšek, prof. Franc Forstnerič, prof. Miran Černe, prof. Pavle Saksida					
<b>Jeziki / Languages:</b>		<b>Predavanja / Lectures:</b> slovenski/Slovene, angleški/English					
		<b>Vaje / Tutorial:</b> slovenski/Slovene, angleški/English					
<b>Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:</b>				<b>Prerequisites:</b>			
<b>Vsebina:</b>				<b>Content (Syllabus outline):</b>			

<p>Definicija Riemannove ploskve. Osnovni primeri. Holomorfne in meromorfne funkcije in preslikave. Topologija Riemannovih ploskev. Krovni prostori in krovne transformacije. Analitično nadaljevanje. Algebraične funkcije. Integracija na Riemannovih ploskvah. Riemannove ploskve kot kompleksne krivulje. Osnovni pojmi teorije snopov.</p> <p>Konstrukcija meromorfni funkcij z L<sup>2</sup>-metodo. Weylova lema. Hilbertov prostor kvadratno integrabilnih form. Meromorfne funkcije in diferenciali. Harmonični in analitični diferenciali. Bilinearne relacije. Divizorji in holomorfni vektorski svežnji. Riemannov-Rochov izrek in uporaba.</p> <p>Možne dodatne vsebine:</p> <p>Odprte Riemannove ploskve. Dirichletov problem. Rungejev aproksimacijski izrek. Mittag-Lefflerjev in Weierstrassov izrek. Riemann-Koebejev uniformizacijski izrek. Riemann-Hilbertov robni problem. Serrejev izrek o dualnosti. Abelov izrek in uporabe. Jacobijev inverzni problem. Kompleksni torusi. Eliptične funkcije. Weierstrassova funkcija.</p>	<p>The notion of a Riemann surface. Basic examples. Holomorphic and meromorphic functions and maps. Topology of Riemann surfaces. Covering spaces and deck transformations. Analytic continuation. Algebraic functions. Integration on Riemann surfaces. Riemann surfaces as complex curves. Basics of sheaf theory.</p> <p>Construction of meromorphic functions by L<sup>2</sup>-method. Weyl lemma. Hilbert space of square integrable forms. Meromorphic functions and differentials. Harmonic and analytic differentials. Bilinear relations. Divisors and holomorphic line bundles. The Riemann-Roch theorem and applications.</p> <p>Other possible topics: Open Riemann surfaces. The Dirichlet problem. The Runge approximation theorem. Theorems of Mittag-Leffler and Weierstrass. Riemann-Koebe uniformization theorem. Riemann-Hilbert boundary value problem. Serre duality. Abel's theorem and applications. Jacobi inverse problem. Complex tori. Elliptic functions. Weierstrass function.</p>
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**Temeljni literatura in viri / Readings:**

H. M. Farkas, I. Kra: Riemann Surfaces, 2nd edition, Springer, New York, 1992.

O. Forster: Lectures on Riemann Surfaces, Springer, New York, 1999.

F. Kirwan: Complex Algebraic Curves, Cambridge Univ. Press, Cambridge, 1992.

B. A. Dubrovin, A. T. Fomenko, S. P. Novikov: Modern Geometry - Methods and Applications III : Introduction to Homology Theory, Springer, New York, 1990.

D. Varolin: Riemann surfaces by way of complex analytic geometry. Amer. Math. Soc., Providence, RI, 2011.

**Cilji in kompetence:**

**Objectives and competences:**

Slušatelj se seznanja z osnovami teorije Riemannovih ploskev in njihovo povezavo s sorodnimi področji matematike kot so kompleksna analiza in algebraična geometrija. Pri tem uporabi znanje iz osnovne analize, algebre in topologije.

V okviru seminarских/projektnih aktivnosti študentje z individualnim delom in predstavitvijo ter delom v skupinah pridobijo izobraževalno komunikacijske in socialne kompetence za prenos znanj in za vodenje (strokovnega skupinskega dela).

Students learn some of the basic concepts and methods of the theory of Riemann surfaces and its connection to related fields of mathematics such as complex analysis and algebraic geometry. Basic methods of analysis, algebra and topology are applied in the course.

With individual presentations and team work interactions within seminar/project activities students acquire communication and social competences for successful team work and knowledge transfer.

### **Predvideni študijski rezultati:**

Znanje in razumevanje:

Spoznanje in razumevanje nekaterih bistvenih osnovnih pojmov teorije Riemannovih ploskev.

Uporaba: Riemannove ploskve so pojavljajo v vrsti matematičnih področij (analitična in algebraična geometrija, diferencialna geometrija, simplektična geometrija), nepogrešljive pa so tudi v mnogih vejah fizike (npr. teorija strun) in širše znanosti. Eliptične krivulje so bistvenega pomena v kriptografiji.

Refleksija: Razumevanje teorije na podlagi primerov. Razvoj sposobnosti uporabe teorije v različnih znanstvenih problemih.

Prenosljive spretnosti – niso vezane le na en predmet: Identifikacija, formulacija in reševanje problemov s pomočjo metod teorije Riemannovih ploskev. Spretnost uporabe domače in tuje literature. Privajanje na samostojno seminarsko predstavitev gradiva.

### **Intended learning outcomes:**

Knowledge and understanding: Understanding of fundamental topics in the theory of Riemann surfaces.

Application: Riemann surfaces appear naturally in many areas of mathematics (e.g. in analytic and algebraic geometry, differential geometry, symplectic geometry and other areas), as well as in several areas of physics (such as string theory) and in other sciences. Elliptic curves are a fundamental tool in cryptography.

Reflection: Understanding the theory on the basis of examples. Acquiring skills in applying the theory to diverse scientific problems.

Transferable skills: The ability to identify, formulate and solve scientific problems using methods of Riemann surface theory. Developing skills of using the domestic and foreign literature. Developing skills of independent presentation of the material.

### **Metode poučevanja in učenja:**

### **Learning and teaching methods:**

predavanja, seminarji, vaje, domače naloge, konzultacije

Lectures, seminar presentations, exercises, homeworks, consultations

Delež (v %) /

**Načini ocenjevanja:**

Weight (in %) /

**Assessment:**

Način (domače naloge, seminarska naloga, ustno izpraševanje):  
domače naloge, seminarska naloga

ustni izpit

Ocene: 1-5 (negativno), 6-10 (pozitivno)  
(po Statutu UL)

50%  
50%

Type (homework, seminar paper, oral exam):

homework and seminar paper

oral exam

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

Miran Černe:

– ČERNE, Miran, FORSTNERIČ, Franc. Embedding some bordered Riemann surfaces in the affine plane. *Mathematical research letters*, ISSN 1073-2780, 2002, vol. 9, no. 5-6, str. 683-696 [COBISS.SI-ID 12391257]

– ČERNE, Miran. Nonlinear Riemann-Hilbert problem for bordered Riemann surfaces. *American journal of mathematics*, ISSN 0002-9327, 2004, vol. 126, no. 1, str. 65-87 [COBISS.SI-ID 12895577]

– ČERNE, Miran, FLORES, Manuel. Quasilinear  $\partial$ -equation on bordered Riemann surfaces. *Mathematische Annalen*, ISSN 0025-5831, 2006, vol. 335, no. 2, str. 379-403 [COBISS.SI-ID 13970777]

Barbara Drinovec Drnovšek:

– DRINOVEC-DRNOVŠEK, Barbara. Discs in Stein manifolds containing given discrete sets. *Mathematische Zeitschrift*, ISSN 0025-5874, 2002, vol. 239, no. 4, str. 683-702 [COBISS.SI-ID 11567449]

– DRINOVEC-DRNOVŠEK, Barbara. Proper discs in Stein manifolds avoiding complete pluripolar sets. *Mathematical research letters*, ISSN 1073-2780, 2004, vol. 11, no. 5-6, str. 575-581 [COBISS.SI-ID 13311065]

– DRINOVEC-DRNOVŠEK, Barbara, FORSTNERIČ, Franc. Holomorphic curves in complex spaces. *Duke mathematical journal*, ISSN 0012-7094, 2007, vol. 139, no. 2, str. 203-254 [COBISS.SI-ID

14351705]

Franc Forstnerič:

– FORSTNERIČ, Franc. Runge approximation on convex sets implies the Oka property. *Annals of mathematics*, ISSN 0003-486X, 2006, vol. 163, no. 2, str. 689-707 [COBISS.SI-ID 13908825]

– FORSTNERIČ, Franc, WOLD, Erlend Fornæss. Bordered Riemann surfaces in  $\mathbb{C}^2$ . *Journal de Mathématiques Pures et Appliquées*, ISSN 0021-7824. [Print ed.], 2009, vol. 91, issue 1, str. 100-114 [COBISS.SI-ID 15395417]

– FORSTNERIČ, Franc, WOLD, Erlend Fornæss. Embeddings of infinitely connected planar domains into  $\mathbb{C}^2$ . *Analysis & PDE*, ISSN 2157-5045, 2013, vol. 6, no. 2, str. 499-514 [COBISS.SI-ID 16645209]

Pavle Saksida:

– SAKSIDA, Pavle. Integrable anharmonic oscillators on spheres and hyperbolic spaces. *Nonlinearity*, ISSN 0951-7715, 2001, vol. 14, no. 5, str. 977-994 [COBISS.SI-ID 10942809]

– SAKSIDA, Pavle. Maxwell-Bloch equations, C Neumann system and Kaluza-Klein theory. *Journal of physics. A, Mathematical and general*, ISSN 0305-4470, 2005, vol. 38, no. 48, str. 10321-10344 [COBISS.SI-ID 13802073]

– SAKSIDA, Pavle. Lattices of Neumann oscillators and Maxwell-Bloch equations. *Nonlinearity*, ISSN 0951-7715, 2006, vol. 19, no. 3, str. 747-768 [COBISS.SI-ID 13932377]