| UČNI NAČRT PREDMETA / COURSE SYLLABUS | |
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| **Predmet:** | Topološka analiza podatkov |
| **Course title:** | Topological data analysis |

| **Študijski program in stopnja**<br>**Study programme and level** | **Študijska smer**<br>**Study field** | **Letnik**<br>**Academic year** | **Semester**<br>**Semester** |
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| Interdisciplinarni magistrski študijski program Računalništvo in matematika | ni smeri | 1 ali 2 | drugi |
| Interdisciplinary Masters study programme Computer Science and Mathematics | none | 1 or 2 | second |

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| **Vrsta predmeta / Course type** | izbirni |
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| **Univerzitetna koda predmeta / University course code:** | M2846 |

| **Predavanja**<br>**Lectures** | **Seminar**<br>**Seminar** | **Vaje**<br>**Tutorial** | **Klinične vaje**<br>**work** | **Druge oblike študija** | **Samost. delo**<br>**Individ. work** | **ECTS** |
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| **45** | **10** | **20** | | | **105** | **6** |

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| **Nosilec predmeta / Lecturer:** | doc. Žiga Virk |

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| **Jeziki / Languages:** | **Predavanja / Lectures:** | slovenski/Slovene, angleški/English |
| | **Vaje / Tutorial:** | slovenski/Slovene, angleški/English |

| **Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:** | **Prerequisites:** |
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| Uspešno opravljene domače naloge in seminarsko-projektne naloge so pogoj za opravljanje izpita. | Successful completion of homework and projects is required for students to approach to a final exam. |
| **Vsebina:** | **Content (Syllabus outline):** |

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| Topologija je področje matematike, ki se ukvarja z analizo oblik in več dimenzionalnih objektov. Topološka analiza podatkov pa je področje med topologijo in računalništvom, ki obravnava in analizira lastnosti oblik zajetih iz podatkov, slik in več dimenzionalnih podatkovnih množic. Ob množici podatkov, ki se neprestano zajemajo, na eni strani in pa vse bolj zmogljivimi računalnškimi sistemi na drugi se razvija tudi vrsta novih algoritmov za analizo in predstavitev, ki uporabljajo čedalje več topoloških pojmov in modelov. Za predstavitev podatkov se uporabljajo grafi in ploskve, triangulacije, simplicialni in celični kompleksi ter mnogoterosti. Za analizo podatkov pa se uporabljajo topološke invariante teh objektov kot so število komponent, fundamentalna grupa, homološke grupe in kohomološki kolobar, Morsova teorija, filtracije in vztrajnost. Takšne invariante se tipično lepo izračunajo in dajejo odgovore na vprašanja kot so, ali je objekt sestavljen iz enega li več kosov, ali ima kakšne luknje in tunele, kakšne značilnosti ima pri različnih resolucijah, kako so posamezni kosi zlepljeni skupaj v celoto... Na drugi strani je na voljo tudi čedalje več hitrih in učinkovitih algoritmov za njihovo računanje.<br><br>Pri predmetu bodo predstavljeni osnovni topološki pojmi in modeli, ki se uporabljajo za predstavitev večdimenzionalnih objektov in prostorov, nekaj njihovih osnovnih številskih in algebraičnih invariant. Poudarek pa bo na uporabi teh modelov in invariant pri analizi in rekonstrukciji objektov iz zajetih podatkov, konfiguracijskih prostorov robotov in mehaničnih sistemov, pri analizi omrežij in v drugih povsem uporabnih domenah. Posamezna teme, ki jih bomo obravnavali, so<br><br>Osnovni pojmi topoloških in metričnih prostorov;<br><br>Grafi in ploskve; | | Topology is the mathematical field dealing with shapes and with modeling and understanding higher dimensional objects. Topologial data analysis is a field between topology and computer science dealing with shapes arising from data, images, and higher dimensional data sets. In view of massive quantities of experimental data on one hand, and available computing power on the other hand, numerous new algorithms and models for qualitative analysis and representation of such data sets using topological models and methods have been developed. Graphs, surfaces, triangulations, simplicial and cell complexes and manifolds are used for data representation and object reconstruction. Topological invariants like the number of components, the fundamental group, hopology groups and the cohomology ring, Morse theory, filtrations and persistence are used for analyzing these models. These invariants are typically computable and give answers to questions like, is the object composed from one or more components, does it have holes and tunnels, which features appear at different resolutions, how do the separate pieces connect into the whole, … On the other hand new algorithms for efficient computation of these invariants are appearing.<br><br>In the course, fundamental topological concepts and methods, which are used in modeling and analyzing higher dimensional objects and spaces, will be introduced. Further, basic numerical and algebraic invariants of the topological models will be explained. Special attention will be given to applications of these methods to analysis of data sets and reconstruction of the underlying objects, configuration spaces of robots and mechanical systems, analysis of networks and other practical problems and domains. We will introduce the following topological concepts and models: |

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| Triangulacije, simplicialni in celični kompleksi;<br><br>Homološke grupe in Bettijeva števila, njihova interpretacija in osnovni algoritmi za njihovo računanje;<br><br>Diskretne Morsove funkcije in njihova uporaba pri analizi podatkov;<br><br>Filtracije in vztrajnost za analizo podatkov pri različnih resolucijah.<br><br>Pri predmetu bo poudarek predvsem na uporabi opisanih topoloških pojmov in algoritmov pri analizi konkretnih podatkovnih množic, problemov in modelov. | Fundamentals of topological and metric spaces<br><br>Graphs and surfaces<br><br>Triangulations, simplicial and cell complexes<br><br>Homology groups and Betti numbers, , their interpretation, and basic algorithms for their computation<br><br>Discrete Morse functions and their application to data analysis and object reconstruction<br><br>Filtrations and persistence for dealing with changing resolutions<br><br>The main part of the course will be devoted to applications of the topological concepts and algorithms in analyzing specific data sets, problems and models. |

**Temeljni literatura in viri / Readings:**

Herbert Edelsbrunner, John Harer: Computational Topology, American Mathematical Society, 2010

Afra J. Zomorodian: Topology for Computing, Cambridge University Press, 2005

Hjelle, Øyvind, Dæhlen, Morten: Triangulations and applications, Springer, 2006

Kevin Knudson: Morse theory, smooth and discrete, World Scientific, 2015

| **Cilji in kompetence:** | **Objectives and competences:** |
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| Cilj predmeta je študentom na razumljiv način predstaviti osnovne pojme algebraične topologije, ki se uporabljajo v računalniških algoritmih pri analizi velikih množic večdimenzionalnih podatkov, pri rekonstrukciji objektov in konfiguracijskih prostorov robotov in mehaničnih sistemov in pri drugih realnih problmemih. Matematični pojmi bodo predstavljni predvsem z uporabnega zornega kota, poudarek bo na konkretnih primerih in računalniških algoritmih. | The aim of this course is to introduce in an informal and intuitive way the basic concepts of algebraic topology which are used in algorithms for analysis of big, possibly higher dimensional data sets, for reconstruction of obects and configuration soaces of robots and mechanical systems and in other practical applications. Mathematical concept will be presented from the point of view of applications, special attention will be given to specific examples and algorithms. |

| **Predvideni študijski rezultati:** | **Intended learning outcomes:** |
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| Po uspešno opravljenem predmetu bo študent:<br>razumel pojem topološke invariante in njenega pomena pri analizi oblike in drugih lastnosti podatkov;<br><br>razumel pojem simplicialnega kompleksa in poznal osnovne algoritme za konstrukcijo simplicialnih<br><br>kompleksov na danih podatkih;<br><br>poznal osnovne računske prijeme in algoritme za računanje Eulerjeve karakteristike, homoloških grup in Bettijevih števil;<br><br>razumel pojem filtracije in vztrajnosti;<br><br>znal pridobljeno znanje uporabiti za konstrukcijo preprostih topoloških algoritmov za analizo podatkovnih množic in rezultate interpretirati. | After completing the course students will:<br>understand the concept of a topological invariant and its role in analyzing shape and other properties of data;<br><br>understand the concept of a simplcial complex and the basic algorithms for constructing simplicial complexes on data sets;<br><br>understand the basic computational approches and algorithms for computing Euler characteristic, homology groups and Betti numbers;<br><br>understand the concepts of filtrations and persistence;<br><br>be able to use the concepts intruduced to construct simple topological algorithms for analysizing data sets and interpret the results. |

| **Metode poučevanja in učenja:** | **Learning and teaching methods:** |
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| Predavanja s podporo avdio-vizualne opreme, predstavitev teoretičnih pojmov in prikaz pomena na konkretnih primerih,<br><br>laboratorijske vaje v računalniški učilnici z ustrezno programsko opremo. Delo posamezno in v skupinah. Velik poudarek na praktičnem delu in na skupinskem reševanju praktičnih problemov. | Combined lecturing with simultaneous use of the blackboard and computer projection explaining the theoretical concepts and specific meaning in specific cases. Lab work in computer-equipped lecture rooms. Individual and work in team. Emphasis on practical problem solving and group work. |

| **Načini ocenjevanja:** | Delež (v %) / Weight (in %) | **Assessment:** |
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| Način (pisni izpit, ustno izpraševanje, naloge, projekt):<br>Sprotno preverjanje (domače naloge, | 60%<br>40% | Type (examination, oral, coursework, project):<br>Continuing (homework, midterm exams, |

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| kolokviji in projektno delo)<br><br>Končno preverjanje (pisni in ustni izpit)<br><br>Ocene: 6-10 pozitivno, 5 negativno<br><br>(v skladu s Statutom UL) | | project work)<br><br>Final: (written and oral exam)<br><br>Grading: 6-10 pass, 5 fail. |

**Reference nosilca / Lecturer's references:**

VIRK, Žiga. Small loop spaces. Topology and its Applications, ISSN 0166-8641, 2010, vol. 157, no. 2, str. 451-455.

VIRK, Žiga. Realizations of countable groups as fundamental groups of compacta. Mediterranean journal of mathematics, 2013, vol. 10, no. 3, str. 1573-1589.

DYDAK, Jerzy, VIRK, Žiga. Preserving coarse properties. Revista matemática complutense, 2016, vol. 29, iss. 1, str. 191-206.

EDELSBRUNNER, Herbert, VIRK, Žiga, WAGNER, Hubert. Smallest enclosing spheres and Chernoff points in Bregman geometry. V: SPECKMANN, Bettina (ur.), TÓTH, Csaba D. (ur.). 34th International Symposium on Computational Geometry : SoCG 2018, June 11-14, 2018, Budapest, Hungary,

VIRK, Žiga. Approximations of 1-dimensional intrinsic persistence of geodesic spaces and their stability. Revista matemática complutense, Jan. 2019, vol. 32, iss. 1, str. 195-213.