UČNI NAČRT PREDMETA / COURSE SYLLABUS (leto / year 2017/18)			
Predmet:	Teorija mere		
Course title:	Measure theory		

Študijski program in stopnja **Study programme and level**	**Študijska smer** **Study field**	**Letnik** **Academic year**	**Semester** **Semester**
Magistrski študijski program Matematika	ni smeri	1 ali 2	prvi ali drugi
Master's study programme Mathematics	none	1 or 2	first or second

Vrsta predmeta / Course type	izbirni temeljni / core elective
Univerzitetna koda predmeta / University course code:	M2110

Predavanja **Lectures**	**Seminar** **Seminar**	**Vaje** **Tutorial**	**Klinične vaje** **work**	**Druge oblike študija**	**Samost. delo** **Individ. work**	**ECTS**
45		**30**			**105**	**6**

Nosilec predmeta / Lecturer:	prof. dr. Roman Drnovšek, prof. dr. Bojan Magajna

Jeziki / Languages:	**Predavanja / Lectures:**	slovenski / Slovene, angleški / English
	Vaje / Tutorial:	slovenski / Slovene, angleški / English

Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:	**Prerequisites:**
Vpis v letnik študija.	Enrolment in the programme.

Vsebina:	**Content (Syllabus outline):**

Mere: σ-algebre, pozitivne mere, zunanje mere, Caratheodoryjev izrek, razširitev mere iz algebre na sigma algebro, Borelove mere na R , Lebesguova mera na R. Merljive funkcije: aproksimacija s stopničastimi funkcijami, načini konvergence funkcijskih zaporedij, izrek Jegorova. Integracija: integral nenegativne funkcije, izrek o monotoni konvergenci, Fatoujeva lema, integral kompleksne funkcije, izrek o dominirani konvergenci, primerjava Riemannovega in Lebesguovega integrala, izrek Jegorova. Produktne mere: konstrukcija produktnih mer, monotoni razredi, Tonellijev in Fubinijev izrek, Lebesguov integral na R^n. Kompleksne mere: predznačene mere, Hahnov in Jordanov razcep, kompleksne mere, variacija mere, absolutna zveznost in vzajemna singularnost, Lebesgue-Radon-Nikodymov izrek. L^p-prostori: neenakosti Jensena, Hölderja in Minkowskega, omejeni linearni funkcionali, dualni prostoir. Integriranje na lokalno kompaktnih prostorih: pozitivni linearni funkcionali na C_c(X), Radonove mere, Rieszov izrek, Lusinov izrek, gostost prostora C_c(X) v L^p-prostorih. Odvajanje mer na R^n : odvajanje mer, absolutno zvezne in funkcije z omejeno totalno variacijo.	Measures: σ-algebras, positive measures, outer measures, Caratheodory's theorem, extension of measures from algebras to σ-algebras, Borel measures on R, Lebesgue measure on R. Measurable functions: approximation by step functions, modes of convergence of sequences of functions, Egoroff's theorem. Integration: integration of nonnegative functions, Lebesgue monotone convergence theorem, Fatou's lemma, integration of complex functions, Lebesgue dominated convergence theorem, comparison with Riemann's integral. Product measures: construction of product measures, monotone classes, Tonelli's and Fubini's theorem, the Lebesgue integral on R^n . Complex measures: signed measures, the Hahn and the Jordan decomposition, complex measures, variation of a measure, absolute continuity and mutual singularity, the Lebesgue-Radon-Nikodym theorem. L^p-spaces: inequalities of Jensen, Hölder and Minkovski, bounded linear functionals, dual spaces. Integration on locally compact spaces: positive linear functionals on C_c(X), Radon measures, Riesz representation theorem, Lusin's theorem, density of C_c(X) in L^p-spaces. Differentiation of measures on R^n : differentiation of measures, absolutely continuous and functions of bounded variation.

Temeljni literatura in viri / Readings:

C. D. Aliprantis, O. Burkinshaw: Principles of Real Analysis, 3rd edition, Academic Press, San Diego, 1998.

R. Drnovšek: Rešene naloge iz teorije mere, DMFA-založništvo, Ljubljana, 2001.

G. B. Folland: Real Analysis : Modern Techniques and Their Applications, 2nd edition, John Wiley & Sons, New York, 1999.

M. Hladnik: Naloge in primeri iz funkcionalne analize in teorije mere, DMFA-založništvo, Ljubljana, 1985.

S. Kantorovitz: Introduction to Modern Analysis, Oxford Univ. Press, 2003.

B. Magajna: Osnove teorije mere, DMFA-založništvo, Ljubljana, 2011.

W. Rudin: Real and Complex Analysis, 3rd edition, McGraw-Hill, New York, 1987.

Cilji in kompetence:

Študent pridobi znanje osnov teorije mere, ki jih potrebuje za razumevanje osnov sodobnega verjetnostnega računa, statistike in funkcionalne analize.

Objectives and competences:

Students acquire basic knowledge of measure theory needed to understand probability theory, statistics and functional analysis.

Predvideni študijski rezultati:

Znanje in razumevanje: Razumevanje osnovnih pojmov teorije mere.

Uporaba: Teorija mere sodi med temeljne predmete na 2. stopnji študija matematike, saj je nujno potrebna za razumevanje osnov sodobnega verjetnostnega računa, statistike in funkcionalne analize. Poleg tega njena uporaba sega tudi v naravoslovje in druga področja znanosti kot na primer ekonomijo.

Refleksija: Razumevanje teorije na podlagi primerov uporabe.

Prenosljive spretnosti – niso vezane le na en predmet: Sposobnost abstraktnega razmišljanja. Spretnost uporabe domače in tuje literature.

Intended learning outcomes:

Knowledge and understanding: understanding basic concepts of measure and integration theory.

Application: measure theory is a part of the basic curriculum of the graduate study of mathematics since it is needed in other areas, for example, in probablity calculus, statistics and functional analysis. It is useful also in other sciences, for example in economy.

Reflection: understanding of the theory on the basis of examples of application.

Transferable skills: Ability to use abstract methods to solve problems. Ability to use a wide range of references and critical thinking.

Metode poučevanja in učenja:	**Learning and teaching methods:**
predavanja, vaje, domače naloge, konzultacije	Lectures, exercises, homeworks, consultations

Načini ocenjevanja:	Delež (v %) / Weight (in %)	**Assessment:**
Način (pisni izpit, ustno izpraševanje, naloge, projekt):		Type (examination, oral, coursework, project):
izpit iz vaj (2 kolokvija ali pisni izpit)		2 midterm exams instead of written exam, written exam
ustni izpit	50%	oral exam
Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)	50%	Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

Reference nosilca / Lecturer's references:

Roman Drnovšek:

DRNOVŠEK, Roman. Spectral inequalities for compact integral operators on Banach function spaces. Mathematical proceedings of the Cambridge Philosophical Society, ISSN 0305-0041, 1992, let. 112, str. 589-598. [COBISS.SI-ID 8169561]

DRNOVŠEK, Roman. On invariant subspaces of Volterra-type operators. Integral equations and operator theory, ISSN 0378-620X, 1997, let. 27, št. 1, str. 1-9. [COBISS.SI-ID 7038553]

DRNOVŠEK, Roman. A generalization of Levinger's theorem to positive kernel operators. Glasgow mathematical journal, ISSN 0017-0895, 2003, vol. 45, part 3, str. 545-555. [COBISS.SI-ID 12825945]

Bojan Magajna:

MAGAJNA, Bojan. Sums of products of positive operators and spectra of Lüders operators. Proceedings of the American Mathematical Society, ISSN 0002-9939, 2013, vol. 141, no. 4, str.

1349-1360. [COBISS.SI-ID 16603481]

MAGAJNA, Bojan. Fixed points of normal completely positive maps on B(H). Journal of mathematical analysis and applications, ISSN 0022-247X. [Print ed.], 2012, vol. 389, iss. 2, str. 1291-1302. [COBISS.SI-ID 16227673]

MAGAJNA, Bojan. Uniform approximation by elementary operators. Proceedings of the Edinburgh Mathematical Society, ISSN 0013-0915, 2009, vol. 52, part 3, str. 731-749. [COBISS.SI-ID 15352921]