

| UČNI NAČRT PREDMETA / COURSE SYLLABUS  |   |   |                                     |                                       |                                    |      |  |  |  |  |
|--|---|---|-------------------------------------|---------------------------------------|------------------------------------|------|--|--|--|--|
| <b>Predmet:</b>  | Slučajni procesi 2                      |   |                                     |                                       |                                    |      |  |  |  |  |
| <b>Course title:</b>   | Stochastic processes 2                  |   |                                     |                                       |                                    |      |  |  |  |  |
| <b>Študijski program in stopnja</b><br><b>Study programme and level</b>          |   | <b>Študijska smer</b><br><b>Study field</b> |                                     | <b>Letnik</b><br><b>Academic year</b> | <b>Semester</b><br><b>Semester</b> |      |  |  |  |  |
| Magistrski študijski program<br>Matematika                                       |   | ni smeri                                    |                                     | 1 ali 2                               | prvi ali drugi                     |      |  |  |  |  |
| Master's study<br>programme Mathematics  |   | none  |                                     | 1 or 2                                | first or second                    |      |  |  |  |  |
| <b>Vrsta predmeta / Course type</b>  |   |   |                                     | izbirni                               |                                    |      |  |  |  |  |
| <b>Univerzitetna koda predmeta / University course code:</b>                     |   |   |                                     | M2520                                 |                                    |      |  |  |  |  |
| Predavanja<br>Lectures   | Seminar<br>Seminar                      | Vaje<br>Tutorial                            | Klinične vaje<br>work               | Druge oblike<br>študija               | Samost. delo<br>Individ.<br>work   | ECTS |  |  |  |  |
| 30   | 15                                      | 30  |                                     |                                       | 105                                | 6    |  |  |  |  |
| <b>Nosilec predmeta / Lecturer:</b>  |   | prof. Janez Bernik, prof. Mihael Perman     |                                     |                                       |                                    |      |  |  |  |  |
| <b>Jeziki /</b><br><b>Languages:</b>   | <b>Predavanja /</b><br><b>Lectures:</b> |   | slovenski/Slovene, angleški/English |                                       |                                    |      |  |  |  |  |
|  | <b>Vaje / Tutorial:</b>                 |   | slovenski/Slovene, angleški/English |                                       |                                    |      |  |  |  |  |
| <b>Pogoji za vključitev v delo oz. za opravljanje<br/>študijskih obveznosti:</b> |   |   |                                     | <b>Prerequisites:</b>                 |                                    |      |  |  |  |  |
|  |   |   |                                     |                                       |                                    |      |  |  |  |  |
| <b>Vsebina:</b>  |   |   |                                     | <b>Content (Syllabus outline):</b>    |                                    |      |  |  |  |  |

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| <p><b>Brownovo gibanje:</b></p> <p>Osnovne lastnosti, obstoj, lastnosti trajektorij, naravna filtracija, čas prvega dotika, markovske lastnosti, krepka lastnost Markova, princip zrcaljenja, pridruženi procesi (proces tekočega supremuma, Brownov most itd.), kvadratična variacija.</p> <p><b>Martingali v zveznem času:</b></p> <p>Filtracije, časi ustavljanja, martingali, izreki o ostavljanju, enakomerna integrabilnost, maksimalne neenakosti, konvergenca martingalov.</p> <p><b>Stohastični integral:</b></p> <p>Stohastični integral glede na Brownovo gibanje, Itova izometrija, zvezni polmartingali, zvezni lokalni martingali, kvadratična variacija in kovariacija, stohastični integral glede na zvezne polmartingale, Itova formula, izrek Girsanova, izrek o reprezentaciji martingalov.</p> | <p><b>Brownian motion:</b></p> <p>Basic properties, existence, path properties, natural filtration, first hitting time, Markov properties, strong Markov property, reflection principle, associated processes (running supremum process, Brownian bridge etc.), quadratic variation.</p> <p><b>Continuous time martingales:</b></p> <p>Filtrations, stopping times, stopping theorems, uniform integrability, maximal inequalities, convergence of martingales.</p> <p><b>Stochastic integral:</b></p> <p>Stochastic integral wrt Brownian motion, Itô isometry, continuous semimartingales, local martingales, quadratic variation and covariation, stochastic integral wrt continuous semimartingales, Itô's formula, Girsanov Theorem, representation of martingales.</p> |
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### **Temeljni literatura in viri / Readings:**

- S. Resnick: Adventures in Stochastic Processes, Birkhäuser Boston, 2002.
- I. Karatzas, S. E. Shreve: Brownian Motion and Stochastic Calculus, 2nd Edition, Springer, 2005.
- M. Yor, D. Revuz: Continuous Martingales and Stochastic Calculus, 2nd Edition, Springer, 2004
- J. M. Steele: Stochastic Calculus and Financial Applications, Springer, New York, 2001.

### **Cilji in kompetence:**

Predmet predstavlja uvod v teorijo slučajnih procesov v zveznem času z zveznimi trajektorijami. Rigorozno obravnava Brownovo gibanje kot osnovni primer in gradnik, vpelje martingale v zveznem času, Itôv stohastični račun in Itovo formulo.

### **Objectives and competences:**

This course is an introduction to the theory of stochastic processes in continuous time with continuous sample paths. It rigorously treats Brownian motion as a basic example and building block, introduces martingales in continuous time, stochastic calculus and

|  |                |
|--|----------------|
|  | Ito's formula. |
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**Predvideni študijski rezultati:**

Znanje in razumevanje: Matematična orodja za strogo obravnavo in uporabo slučajnih procesov.

Uporaba:

Osnova za modeliranje v mnogih vejah matematike in njene uporabe.

Refleksija:

Vsebina predmeta pomaga za nazaj poglobiti razumevanje konceptov verjetnosti, koncepta odvisnosti in časa.

Prenosljive spretnosti – niso vezane le na en predmet:

Spretnosti so prenosljive na druga področja matematičnega modeliranja, še najbolj pa je predmet pomemben zaradi svoje neposredne uporabnosti pri finančnem modeliranju.

**Intended learning outcomes:**

Knowledge and understanding:

Mathematical tools for rigorous treatment and applications of stochastic processes.

Application:

Basic tools for modelling in many branches of Mathematics and its applications.

Reflection:

The contents of the course help in retrospect to deepen the understanding of the concepts of probability, dependence and time.

Transferable skills:

The skills acquired are transferable to other areas of mathematical modelling, in particular it is immediately applicable to financial models.

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations

Delež (v %) /

| <b>Načini ocenjevanja:</b>   | <b>Weight (in %)</b> | <b>Assessment:</b>  |
|--|----------------------|---|
| <p>Način (pisni izpit, ustno izpraševanje, naloge, projekt):<br/>pisni izpit</p> <p>Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)</p> | 100%                 | <p>Type (examination, oral, coursework, project):<br/>written exam</p> <p>Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)</p> |

#### **Reference nosilca / Lecturer's references:**

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| Janez Bernik:  |
| <ul style="list-style-type: none"> <li>- BERNIK, Janez, MASTNAK, Mitja, RADJAVI, Heydar. Realizing irreducible semigroups and real algebras of compact operators. <i>Journal of mathematical analysis and applications</i>, ISSN 0022-247X. [Print ed.], 2008, vol. 348, no. 2, str. 692-707. [COBISS.SI-ID 14899289]</li> </ul> |
| <ul style="list-style-type: none"> <li>- BERNIK, Janez, MASTNAK, Mitja, RADJAVI, Heydar. Positivity and matrix semigroups. <i>Linear Algebra and its Applications</i>, ISSN 0024-3795. [Print ed.], 2011, vol. 434, iss. 3, str. 801-812 [COBISS.SI-ID 15745625]</li> </ul>  |
| <ul style="list-style-type: none"> <li>- BERNIK, Janez, MARCOUX, Laurent W., RADJAVI, Heydar. Spectral conditions and band reducibility of operators. <i>Journal of the London Mathematical Society</i>, ISSN 0024-6107, 2012, vol. 86, no. 1, str. 214-234. [COBISS.SI-ID 16357721]</li> </ul>                                  |
| Mihael Perman:   |
| <ul style="list-style-type: none"> <li>- PERMAN, Mihael, WELLNER, Jon A. On the distribution of Brownian areas. <i>Annals of applied probability</i>, ISSN 1050-5164, 1996, let. 6, št. 4, str. 1091-1111 [COBISS.SI-ID 7101017]</li> </ul>  |
| <ul style="list-style-type: none"> <li>- PERMAN, Mihael, PITMAN, Jim, YOR, Marc. Size-biased sampling of Poisson processes and excursions. <i>Probability theory and related fields</i>, ISSN 0178-8051, 1992, 92, no. 1, str. 21-39 [COBISS.SI-ID 12236377]</li> </ul>  |