Expander graphs

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Content:

Expander graphs are a type of highly connected and at the same time sparse graphs. They come up in numerous contexts in computer science and mathematics. Among these are error reduction in probabilistic algorithms, sieving methods in number theory, fast generation of pseudorandom elements in groups, graph and knot embedding problems, etc.

We will introduce three equivalent definitions of expander graphs: combinatorial, algebraic, and probabilistic. A large part of the course will be dedicated to the (nontrivial!) problem of explicitly constructing examples of expander graphs. Most of these known constructions are highly symmetric graphs coming from groups of Lie type, such as SL2(Z) and its finite quotients. We will show how expansion in these groups is related to Kazhdan's property (T) and quasirandomness from representation theory, approximate subgroups from nonabelian additive combinatorics, and the rate of growth of subsets under multiplication from geometric group theory with some basic algebraic geometry. We will follow the influential papers of Helfgott [2] and Bourgain–Gamburd [1].

Time permitting, we will discuss in more detail some of the applications of expander graphs.

Prerequisites

Basic undergraduate knowledge of graph theory, group theory, probability, and functional analysis. Some very basic knowledge of representation theory and algebraic geometry will also be useful.

References

[1] Jean Bourgain and Alex Gamburd. "Uniform expansion bounds for Cayley graphs of $SL2(Fp)$". In: *Annals of Mathematics* (2008), pp. 625–642.

[2] Harald Andrés Helfgott. "Growth and generation in $SL2(Z/pZ)$". In:*Annals of Mathematics* (2008), pp. 601–623.

[3] Shlomo Hoory, Nathan Linial, and Avi Wigderson. "Expander graphs and their applications". In: *Bulletin of the American Mathematical Society* 43.4 (2006), pp. 439– 561.

[4] Emmanuel Kowalski. *An introduction to expander graphs.* Société Mathématique de France, 2019.

[5] Alexander Lubotzky. "Expander graphs in pure and applied mathematics". In: *Bulletin of the American Mathematical Society* 49.1 (2012), pp. 113–162.

[6] Terence Tao. *Expansion in finite simple groups of Lie type.* Vol. 164. American Mathematical Soc., 2015.

Semester: second