Groups acting on graphs

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Content:
A symmetry of a graph is a permutation of the vertices of the graph that preserves the adjacency, that is, maps pairs of adjacent vertices to pairs of adjacent vertices. The set of symmetries together with the operation of composition forms a group, called the automorphism group of the graph. For example, the automorphism group of a complete graph on n vertices is the symmetric group of degree n, while the automorphism group of a cycle on n vertices is the dihedral group of order 2n. The size of the automorphism group and the way it acts on the graph can be viewed as a measures of symmetry of the graph—larger the group, more symmetric is the graph. Similarly, if the group is transitive on the vertices (that is, any vertex can be mapped to any other vertex by a symmetry), then all the vertices 'look the same' and the graph is considered very symmetric. Since symmetry is one of the central notions in mathematics, questions about groups and their actions on graphs is a very important and well developed field of mathematics, which lies at the intersection of algebra and graph theory. In this course we will learn some classical results from this field with the emphasis on the results pertaining to the finite graphs the automorphism group of which acts transitively on vertices, edges and/or directed edges of the graph, including the famous Tutte's result about the order of the automorphism group of of cubic arc-transitive graphs. We will assume that students are familiar with basic notions and results from group theory (such as notions of homomorphisms, normal subgroups, quotients, isomorphism theorems, and a bit more). We will, however, give a very short introduction to some of these algebraic topics at the beginning of the course, including some basic results about permutation groups. Basic understanding of graphs is of course also expected. The final mark will be based on a written test and an oral exam following it.

Semester: first