

Representation Theory

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Course description. Linear representation of a group G is a homomorphism from G into the group of invertible linear maps on a vector space V . Such a map therefore attempts to represent the elements of G as symmetries of the vector space V . The concepts of representation theory turn out to be fundamental in various areas of current mathematics, mathematical physics, and chemistry. Representation theory of the symmetric groups plays a prominent role in combinatorics.

We will start with the simplest case of representation theory: the linear representations of finite groups in finite-dimensional complex vector spaces. This is also historically the first case that was studied in depth. It is a rich theory with many important applications. It can also serve as a guide to many generalizations: various facts, which are extremely elementary for finite groups, remain valid, when properly framed, for important classes of infinite groups. Among these, the compact topological groups are undoubtedly those closest to finite groups, and we will also briefly mention algebraic groups.

Representation theory of finite groups offers possibilities to address some of the questions using computational tools. We will look at these during this course.

One can also replace groups by other objects: we will briefly look at associative algebras and Lie algebras, and their representations.

Syllabus. Linear representations of finite groups. Representation theory of compact groups. Representations of algebras. Applications.

Prerequisites. Basic knowledge of group theory, linear algebra, and topology.

REFERENCES

- [1] The GAP Group, *GAP – Groups, Algorithms, and Programming, Version 4.10.1*; 2019, (<https://www.gap-system.org>).
- [2] E. Kowalski, *An introduction to the representation theory of groups*, Graduate Studies in Mathematics, 155, AMS, Providence, 2014.
- [3] K. Lux, and H. Pahlings, *Representations of groups: A computational approach*, Cambridge Studies in Advanced Mathematics, 124, Cambridge University Press, 2010.
- [4] J.-P. Serre, *Linear representations of finite groups*, Graduate Texts in Mathematics, 42, Springer-Verlag, New York, 1977.