

UČNI NAČRT PREDMETA / COURSE SYLLABUS (leto / year 2016/17)						
<b>Predmet:</b>		Analiza na mnogoterostih				
<b>Course title:</b>		Analysis on manifolds				
<b>Študijski program in stopnja</b> Study programme and level		<b>Študijska smer</b> Study field		<b>Letnik</b> Academic year	<b>Semester</b> Semester	
Magistrski študijski program Finančna matematika		ni smeri		1 ali 2	prvi ali drugi	
Master's study programme Financial Mathematics		none		1 or 2	first or second	
<b>Vrsta predmeta / Course type</b>				izbirni / elective		
<b>Univerzitetna koda predmeta / University course code:</b>				M2308		
<b>Predavanja</b> Lectures	<b>Seminar</b> Seminar	<b>Vaje</b> Tutorial	<b>Klinične vaje</b> work	<b>Druge oblike študija</b>	<b>Samost. delo</b> Individ. work	<b>ECTS</b>
45		30			105	6
<b>Nosilec predmeta / Lecturer:</b>		prof. dr. Franc Forstnerič, prof. dr. Janez Mrčun, prof. dr. Pavle Saksida				
<b>Jeziki / Languages:</b>	<b>Predavanja / Lectures:</b>	slovenski / Slovene, angleški / English				
	<b>Vaje / Tutorial:</b>	slovenski / Slovene, angleški / English				
<b>Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:</b>				<b>Prerequisites:</b>		
Vpis v letnik študija.				Enrolment in the programme.		
<b>Vsebina:</b>				<b>Content (Syllabus outline):</b>		

<p>Definicija gladke mnogoterosti in preslikave. Osnovne konstrukcije in primeri. Diferencial preslikave. Tangentni sveženj in tangentna preslikava. Mnogoterosti z robom. Delovanje grupe na mnogoterosti. Krovne in kvocientne mnogoterosti. Svežnji in vektorski svežnji. Imerzije in submerzije. Podmnogoterosti. Vložitve mnogoterosti v evklidske prostore.</p> <p>Vektorska polja kot dinamični sistemi. Tok vektorskega polja. Komutator vektorskih polj. Frobeniusov izrek. Izrek o obstoju cevaste okolice. Indeks kritične točke vektorskega polja. Poincaré-Hopfov izrek.</p> <p>Liejeve grupe. Eksponentna preslikava. Invariantna vektorska polja. Liejeva algebra. Adjungirana reprezentacija.</p> <p>Sardov izrek. Thomov izrek o transverzalnosti. Presečno število podmnogoterosti. Morsejeve funkcije.</p> <p>Možne dodatne vsebine:</p> <p>Diferencialne forme in integracija. Stokesov izrek. De Rhamova kohomologija. Poincaréjeva dualnost. Eulerjev razred in Thomov razred.</p> <p>Riemannove mnogoterosti. Volumska forma in integracija. Hodgev *-operator. Laplaceov operator. Harmonične forme. Hodgejeva dekompozicija.</p>	<p>The notion of a smooth manifold and map. Basic constructions and examples. The differential. The tangent bundle and the tangent map. Manifolds with boundary. Group actions on manifolds. Covering and quotient manifolds. Fiber bundles and vector bundles.</p> <p>Immersion and submersions. Submanifolds. Embedding manifolds to Euclidean spaces.</p> <p>Vector fields as dynamical systems Flows. Commutator of vector fields. The theorem of Frobenius. The tubular neighborhood theorem. Index of a critical point of a vector field. The Poincaré-Hopf theorem.</p> <p>Lie groups. The exponential map. Invariant vector fields. The Lie algebra of a Lie group. The adjoint representation.</p> <p>Sard's theorem. The Thom transversality theorem. The intersection number of submanifolds. Morse functions.</p> <p>Other possible topics:</p> <p>Differential forms and integration. Stokes' theorem. De Rham cohomology. Poincaréjeva dualnost. Eulerjev and Thomov class.</p> <p>Riemannian manifolds. Volume form and integration. The Hodge *-operator. Laplace operator. Harmonic forms. Hodge decomposition.</p>
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### Temeljni literatura in viri / Readings:

- W. M. Boothby: An Introduction to Differentiable Manifolds and Riemannian Geometry, 2nd edition, Academic Press, Orlando, 1986.
- V. Guillemin, A. Pollack: Differential Topology, Prentice Hall, Englewood Cliffs, 1974.
- M. W. Hirsch: Differential Topology, Springer, New York, 1997.
- M. Spivak: Calculus on Manifolds, W. A. Benjamin, New York-Amsterdam, 1965.

F. W. Warner: Foundations of Differentiable Manifolds and Lie Groups, Springer, New York-Berlin, 1983.

#### **Cilji in kompetence:**

Slušatelj se seznani z osnovami teorije gladkih mnogoterosti in njihovo povezavo s sorodnimi področji matematike kot so analitična in algebraična geometrija, teorija Riemannovih ploskev, teorija Liejevih grup in druga. Pri tem uporabi znanje iz osnovne analize, algebre in topologije.

#### **Objectives and competences:**

Students learn some of the main basic concepts and methods of the theory of smooth manifolds and its connection to related fields of mathematics such as analytic and algebraic geometry, the theory of Lie groups, the theory of Riemann surfaces, etc. Basic methods of analysis, algebra and topology are applied in the course.

#### **Predvideni študijski rezultati:**

Znanje in razumevanje: Metode matematične analize, algebre in topologije, ki jih je študent spoznal na prvi stopnji študija, se obravnavajo in uporabijo v splošnejšem kontekstu gladkih mnogoterosti. Uporaba: Teorija mnogoterosti je ena najbolj interdisciplinarnih področij sodobne matematike in je osnova vrsti področij kot so analitična, algebraična in diferencialna geometrija, teorija Liejevih grup, teorija Riemannovih ploskev, dinamika, itd.

Mnogoterosti so nepogrešljivo orodje v naravoslovju in tehniki.  
Refleksija: Razumevanje teorije na podlagi primerov. Razvoj sposobnosti uporabe teorije v različnih problemih znanosti in tehnike.

Prenosljive spretnosti – niso vezane le na en predmet: Identifikacija, formulacija in reševanje problemov s pomočjo metod teorije gladkih mnogoterosti. Spretnost uporabe domače in tuje literature.

#### **Intended learning outcomes:**

Knowledge and understanding: Methods of mathematical analysis, algebra and topology are applied and further developed in the context of smooth manifolds.

Application: The theory of smooth manifolds is one of the most interdisciplinary areas of modern mathematics. It is a basis of a number of areas such as analytic, algebraic and differential geometry, the theory of Lie groups, the theory of Riemann surfaces, dynamics, etc. Manifolds are a major tool in natural and technical sciences.

Reflection: Understanding the theory on the basis of examples. Acquiring skills in applying the theory to diverse scientific problems.

Transferable skills: The ability to identify, formulate and solve scientific problems using methods of smooth manifolds. Developing skills of using the domestic and foreign literature.

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**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations

**Načini ocenjevanja:**

Delež (v %) /

Weight (in %)

**Assessment:**

Način (domače naloge, pisni izpit, ustno izpraševanje):

Domače naloge in/ali pisni izpit

ustni izpit

Ocene: 1-5 (negativno), 6-10 (pozitivno)  
(po Statutu UL)

50%

50%

Type (homework, written exam, oral exam):

Homework and/or written exam

oral exam

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

Franc Forstnerič:

FORSTNERIČ, Franc. Runge approximation on convex sets implies the Oka property. *Annals of mathematics*, ISSN 0003-486X, 2006, vol. 163, no. 2, str. 689-707. [COBISS.SI-ID 13908825]

FORSTNERIČ, Franc. Noncritical holomorphic functions on Stein manifolds. *Acta mathematica*, ISSN 0001-5962, 2003, vol. 191, no. 2, str. 143-189. [COBISS.SI-ID 13138009]

FORSTNERIČ, Franc. Manifolds of holomorphic mappings from strongly pseudoconvex domains. *The Asian journal of mathematics*, ISSN 1093-6106, 2007, vol. 11, no. 1, str. 113-126. [COBISS.SI-ID 14352473]

Janez Mrčun:

MOERDIJK, Ieke, MRČUN, Janez. *Introduction to foliations and Lie groupoids*, (Cambridge studies in advanced mathematics, 91). Cambridge, UK: Cambridge University Press, 2003. IX, 173 str., ilustr.

ISBN 0-521-83197-0. [COBISS.SI-ID 12683097]

MOERDIJK, Ieke, MRČUN, Janez. On integrability of infinitesimal actions. American journal of mathematics, ISSN 0002-9327, 2002, vol. 124, no. 3, str. 567-593. [COBISS.SI-ID 11700057]

MRČUN, Janez. Functoriality of the bimodule associated to a Hilsum-Skandalis map. K-theory, ISSN 0920-3036, 1999, let. 18, št. 3, str. 235-253. [COBISS.SI-ID 9163353]

Pavle Saksida:

SAKSIDA, Pavle. Integrable anharmonic oscillators on spheres and hyperbolic spaces. Nonlinearity, ISSN 0951-7715, 2001, vol. 14, no. 5, str. 977-994. [COBISS.SI-ID 10942809]

SAKSIDA, Pavle. Lattices of Neumann oscillators and Maxwell-Bloch equations. Nonlinearity, ISSN 0951-7715, 2006, vol. 19, no. 3, str. 747-768. [COBISS.SI-ID 13932377]

SAKSIDA, Pavle. On zero-curvature condition and Fourier analysis. Journal of physics. A, Mathematical and theoretical, ISSN 1751-8113, 2011, vol. 44, no. 8, 085203 (19 str.). [COBISS.SI-ID 15909465]