

UČNI NAČRT PREDMETA / COURSE SYLLABUS (leto / year 2017/18)											
Predmet:	Kompleksna analiza										
Course title:	Complex analysis										
Študijski program in stopnja Study programme and level	Študijska smer Study field		Letnik Academic year	Semester Semester							
Magistrski študijski program Finančna matematika	ni smeri		1 ali 2	prvi ali drugi							
Master's study programme Financial Mathematics	none		1 or 2	first or second							
Vrsta predmeta / Course type	izbirni / elective										
Univerzitetna koda predmeta / University course code:	M2113										
Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS					
30	15	30			105	6					
Nosilec predmeta / Lecturer:	prof. dr. Miran Černe, prof. dr. Barbara Drinovec Drnovšek, prof. dr. Franc Forstnerič, prof. dr. Jasna Prezelj										
Jeziki / Languages:	Predavanja / Lectures:	slovenski / Slovene, angleški / English									
	Vaje / Tutorial:	slovenski / Slovene, angleški / English									
Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:	Prerequisites:										
Vpis v letnik študija.	Enrolment in the programme.										
Vsebina:	Content (Syllabus outline):										

<p>Cauchyjeva integralska formula za holomorfne in neholomorfne funkcije. Rešitev nehomogene debar enačbe na ravninskih območjih s Cauchyjevim integralom.</p> <p>Schwarzova lema. Avtomorfizmi diska.</p> <p>Konveksne funkcije. Hadamardov izrek o treh krožnicah. Phragmen-Lindelöfov izrek.</p> <p>Kompaktnost in konvergenca v prostoru holomorfnih funkcij. Normalne družine. Montelov izrek. Hurwitzev izrek. Riemannov upodobitveni izrek.</p> <p>Koebejev izrek. Blochov izrek. Izrek Landaua, Picardovi izreki. Schottkyjev izrek.</p> <p>Konvergenca produktov. Weierstrassov faktorizacijski izrek. Rungejev izrek o aproksimaciji z racionalnimi funkcijami. Mittag-Lefflerjev izrek o konstrukciji funkcije z danimi glavnimi deli. Izrek o interpolaciji s holomorfnjo funkcijo na diskretni množici.</p> <p>Schwarzev princip zrcaljenja. Analitično nadaljevanje vzdolž poti. Monodromijski izrek. Kompletna analitična funkcija. Snop zarodkov holomorfnih funkcij. Pojem Riemannove ploskve.</p> <p>Druge možne vsebine: Harmonične in subharmonične funkcije. Poissonovo jedro in rešitev Dirichletovega problema na krogu. Lastnosti Poissonovega integrala in povezava s Cauchyjevim integralom. Mergelyanov izrek. Cele funkcije. Rast in red cele funkcije. Hadamardov izrek o faktorizaciji.</p>	<p>Cauchy integral formula for holomorphic and non holomorphic functions. Solution to the non homogeneous debar equation on planar domains using Cauchy integral.</p> <p>Schwarz lemma. Automorphisms of the unit disc.</p> <p>Convex functions. Hadamard three-circle theorem. Phragmen-Lindelöf theorem.</p> <p>Compactness and convergence in the space of holomorphic functions. Normal families. Montel's theorem. Hurwitz's theorem. Riemann mapping theorem.</p> <p>Koebe's theorem. Bloch's theorem. Landau's theorem, Picard's theorem. Schottky's theorem.</p> <p>Product convergence. Weierstrass factorization theorem. Runge's theorem on approximation by rational functions. Mittag-Leffler's theorem on existence of holomorphic functions with prescribed principal parts. Interpolation by holomorphic functions on discrete sets.</p> <p>Schwarz reflection principle. Analytic continuation along path. Monodromy theorem. Complete analytic function. Sheaf of germs of analytic functions. Riemann surface.</p> <p>Other possible topics: Harmonic and subharmonic functions. Poisson kernel and the solution of the Dirichlet problem on the disc. Properties of Poisson integral and connection to the Cauchy integral. Mergelyan theorem. Entire functions. The genus and the order of entire function. Hadamard factorization theorem.</p>
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### Temeljni literatura in viri / Readings:

- L. Ahlfors: Complex Analysis, 3rd edition, McGraw-Hill, New York, 1979.
- C. A. Berenstein, R. Gay: Complex Analysis and Special Topics in Harmonic Analysis, Springer, New York, 1995.
- J. B. Conway: Functions of One Complex Variable I, 2nd edition, Springer, New York-Berlin, 1995.
- R. Narasimhan, Y. Nievergelt: Complex Analysis in One Variable, 2nd edition, Birkhäuser, Boston, 2001.
- W. Rudin: Real and Complex Analysis, 3rd edition, McGraw-Hill, New York, 1987.
- T. Gamelin: Complex analysis, Springer-Verlag, New York, 2001.

#### **Cilji in kompetence:**

Slušatelj spozna nekatere vsebine teorije holomorfnih funkcij ene kompleksne spremenljivke. Pri tem uporabi znanje iz osnovne analize in topologije.  
V okviru seminarskih/projektnih aktivnosti študentje z individualnim delom in predstavljivo ter delom v skupinah pridobijo izobraževalno komunikacijske in socialne kompetence za prenos znanj in za vodenje (strokovnega skupinskega dela).

#### **Objectives and competences:**

Students learn some basic concepts of theory of functions of one complex variable. Elementary methods of analysis and topology are applied.  
With individual presentations and team work interactions within seminar/project activities students acquire communication and social competences for successful team work and knowledge transfer.

#### **Predvideni študijski rezultati:**

Znanje in razumevanje: Razumevanje nekaterih bistvenih pojmov in rezultatov teorije holomorfnih funkcij.  
Uporaba: V ostalih delih matematične analize in geometrije, uporaba konformnih preslikav pri reševanju problemov iz fizike in mehanike.  
Refleksija: Razumevanje teorije na podlagi primerov in uporabe.  
Prenosljive spretnosti – niso vezane le na en predmet: Identifikacija, formulacija in reševanje matematičnih in nematematičnih problemov s pomočjo metod kompleksne analize. Spretnost uporabe domače in tujе literature. Privajanje

#### **Intended learning outcomes:**

Knowledge and understanding: Understanding some of the fundamental topics and techniques of complex analysis.  
Application: Applications lie mainly in other parts of mathematical analysis and geometry. Conformal maps are applied to solving problems in physics and mechanics.  
Reflection: Understanding the theory on the basis of examples and applications.  
Transferable skills: The ability to identify, formulate and solve mathematical and non mathematical problems using methods of complex analysis. Acquiringn skills in using

na samostojno seminarsko predstavitev gradiva.

domestic and foreign literature. Developing the skills of independent presentation of the material in the form of seminar lectures.

**Metode poučevanja in učenja:**

predavanja, vaje, seminar, domače naloge, konzultacije

**Learning and teaching methods:**

Lectures, exercises, seminar, homeworks, consultations

Delež (v %) /

Weight (in %)    **Assessment:**

Način (domače naloge, seminarska naloga, ustno izpraševanje):  domače naloge, seminarska naloga  ustni izpit  Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)	50%  50%	Type (homework, seminar paper, oral exam, coursework, project):  homework and seminar paper  oral exam  Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)
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**Reference nosilca / Lecturer's references:**

Miran Černe:

ČERNE, Miran, ZAJEC, Matej. Boundary differential relations for holomorphic functions on the disc. Proceedings of the American Mathematical Society, ISSN 0002-9939, 2011, vol. 139, no. 2, str. 473-484. [COBISS.SI-ID 15710553]

ČERNE, Miran, FLORES, Manuel. Generalized Ahlfors functions. Transactions of the American Mathematical Society, ISSN 0002-9947, 2007, vol. 359, no. 2, str. 671-686. [COBISS.SI-ID 14227801]

ČERNE, Miran, FLORES, Manuel. Quasilinear  $\overline{\partial}$ -equation on bordered Riemann surfaces. Mathematische Annalen, ISSN 0025-5831, 2006, vol. 335, no. 2, str. 379-403. [COBISS.SI-ID 13970777]

Barbara Drinovec Drnovšek:

DRINOVEC-DRNOVŠEK, Barbara, FORSTNERIČ, Franc. The Poletsky-Rosay theorem on singular complex spaces. *Indiana University mathematics journal*, ISSN 0022-2518, 2012, vol. 61, no. 4, str. 1407-1423. [COBISS.SI-ID 16679257]

DRINOVEC-DRNOVŠEK, Barbara, FORSTNERIČ, Franc. Holomorphic curves in complex spaces. *Duke mathematical journal*, ISSN 0012-7094, 2007, vol. 139, no. 2, str. 203-254. [COBISS.SI-ID 14351705]

DRINOVEC-DRNOVŠEK, Barbara. On proper discs in complex manifolds. *Annales de l'Institut Fourier*, ISSN 0373-0956, 2007, t. 57, fasc. 5, str. 1521-1535. [COBISS.SI-ID 14379865]

Franc Forstnerič:

FORSTNERIČ, Franc, WOLD, Erlend Fornæss. Embeddings of infinitely connected planar domains into  $C^2$ . *Analysis & PDE*, ISSN 2157-5045, 2013, vol. 6, no. 2, str. 499-514. [COBISS.SI-ID 16645209]

FORSTNERIČ, Franc. Runge approximation on convex sets implies the Oka property. *Annals of mathematics*, ISSN 0003-486X, 2006, vol. 163, no. 2, str. 689-707. [COBISS.SI-ID 13908825]

FORSTNERIČ, Franc. Noncritical holomorphic functions on Stein manifolds. *Acta mathematica*, ISSN 0001-5962, 2003, vol. 191, no. 2, str. 143-189. [COBISS.SI-ID 13138009]

Jasna Prezelj:

FORSTNERIČ, Franc, PREZELJ-PERMAN, Jasna. Oka's principle for holomorphic submersions with sprays. *Mathematische Annalen*, ISSN 0025-5831, 2002, band 322, heft 4, str. 633-666. [COBISS.SI-ID 11554649]

PREZELJ-PERMAN, Jasna. Interpolation of embeddings of Stein manifolds on discrete sets. *Mathematische Annalen*, ISSN 0025-5831, 2003, band 326, heft 2, str. 275-296. [COBISS.SI-ID 12518489]

PREZELJ-PERMAN, Jasna. Weakly regular embeddings of Stein spaces with isolated singularities. *Pacific journal of mathematics*, ISSN 0030-8730, 2005, vol. 220, no. 1, str. 141-152. [COBISS.SI-ID 13910873]