

UČNI NAČRT PREDMETA / COURSE SYLLABUS (leto / year 2016/17)											
Predmet:	Računska topologija										
Course title:	Computational topology										
Študijski program in stopnja Study programme and level	Študijska smer Study field		Letnik Academic year	Semester Semester							
Interdisciplinarni magistrski študijski program Računalništvo in matematika	ni smeri		1 ali 2	drugi							
Interdisciplinary Master's study programme Computer Science and Mathematics	none		1 or 2	second							
Vrsta predmeta / Course type	izbirni / elective										
Univerzitetna koda predmeta / University course code:	M2831										
Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS					
45		30			105	6					
Nosilec predmeta / Lecturer:	prof. dr. Neža Mramor-Kosta										
Jeziki / Languages:	Predavanja / Lectures:	slovenski / Slovene, angleški / English									
	Vaje / Tutorial:	slovenski / Slovene, angleški / English									
Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:	Prerequisites:										
Vpis v letnik študija.	Enrolment in the programme.										
Vsebina:	Content (Syllabus outline):										

Topologija je področje matematike, ki se ukvarja z analizo oblik in več dimenzionalnih objektov. Topološka analiza podatkov pa je področje med topologijo in računalništvom, ki obravnava in analizira lastnosti oblik zajetih iz podatkov, slik in več dimenzionalnih podatkovnih množic. Ob množici podatkov, ki se neprestano zajemajo, na eni strani in pa vse bolj zmogljivimi računalnškimi sistemi na drugi se razvija tudi vrsta novih algoritmov za analizo in predstavitev, ki uporabljajo čedalje več topoloških pojmov in modelov. Za predstavitev podatkov se uporabljajo grafi in ploskve, triangulacije, simplicialni in celični kompleksi ter mnogoterosti. Za analizo podatkov pa se uporabljajo topološke invariante teh objektov kot so število komponent, fundamentalna grupa, homološke grupe in kohomološki kolobar, Morsova teorija, filtracije in vztrajnost. Takšne invariante se tipično lepo izračunajo in dajejo odgovore na vprašanja kot so, ali je objekt sestavljen iz enega li več kosov, ali ima kakšne luknje in tunele, kakšne značilnosti ima pri različnih resolucijah, kako so posamezni kosi zlepjeni skupaj v celoto... Na drugi strani je na voljo tudi čedalje več hitrih in učinkovitih algoritmov za njihovo računanje.

Pri predmetu bodo predstavljeni osnovni topološki pojmi in modeli, ki se uporabljajo za predstavitev večdimenzionalnih objektov in prostorov, nekaj njihovih osnovnih številskih in algebraičnih invariant. Poudarek pa bo na uporabi teh modelov in invariant pri analizi in rekonstrukciji objektov iz zajetih podatkov, konfiguracijskih prostorov robotov in mehaničnih sistemov, pri analizi omrežij in v drugih povsem uporabnih domenah.

Posamezna teme, ki jih bomo obravnavali, so

Osnovni pojmi topoloških in metričnih prostorov

Topology is the mathematical field dealing with shapes and with modeling and understanding higher dimensional objects. Topological data analysis is a field between topology and computer science dealing with shapes arising from data, images, and higher dimensional data sets. In view of massive quantities of experimental data on one hand, and available computing power on the other hand, numerous new algorithms and models for qualitative analysis and representation of such data sets using topological models and methods have been developed. Graphs, surfaces, triangulations, simplicial and cell complexes and manifolds are used for data representation and object reconstruction. Topological invariants like the number of components, the fundamental group, homology groups and the cohomology ring, Morse theory, filtrations and persistence are used for analyzing these models. These invariants are typically computable and give answers to questions like, is the object composed from one or more components, does it have holes and tunnels, which features appear at different resolutions, how do the separate pieces connect into the whole, ... On the other hand new algorithms for efficient computation of these invariants are appearing.

In the course, fundamental topological concepts and methods, which are used in modeling and analyzing higher dimensional objects and spaces, will be introduced. Further, basic numerical and algebraic invariants of the topological models will be explained. Special attention will be given to applications of these methods to analysis of data sets and reconstruction of the underlying objects, configuration spaces of robots and mechanical systems, analysis of networks and other practical problems and domains. We will introduce the following topological concepts and models:

Grafi in ploskve	Fundamentals of topological and metric spaces
Triangulacije, simplicialni in celični kompleksi	Graphs and surfaces
Homološke grupe in Bettijeva števila, njihova interpretacija in osnovni algoritmi za njihovo računanje	Triangulations, simplicial and cell complexes
Diskretne Morseove funkcije in njihova uporaba pri analizi podatkov	Homology groups and Betti numbers, , their interpretation, and basic algorithms for their computation
Filtracije in vztrajnost za analizo podatkov pri različnih resolucijah	Discrete Morse functions and their application to data analysis and object reconstruction
Pri predmetu bo poudarek predvsem na uporabi opisanih topoloških pojmov in algoritmov pri analizi konkretnih podatkovnih množic, problemov in modelov.	Filtrations and persistence for dealing with changing resolutions
	The main part of the course will be devoted to applications of the topological concepts and algorithms in analyzing specific data sets, problems and models.

#### **Temeljni literatura in viri / Readings:**

- Herbert Edelsbrunner, John Harer: Computational Topology, American Mathematical Society, 2010
- Afra J. Zomorodian: Topology for Computing, Cambridge University Press, 2005
- Hjelle, Øyvind, Dæhlen, Morten: Triangulations and applications, Springer, 2006
- Kevin Knudson: Morse theory, smooth and discrete, World Scientific, 2015

#### **Cilji in kompetence:**

Cilj predmeta je študentom na razumljiv način predstaviti osnovne pojme algebraične topologije, ki se uporabljajo v računalniških algoritmih pri analizi velikih množic večdimensionalnih podatkov, pri rekonstrukciji objektov in konfiguracijskih prostorov robotov in mehaničnih sistemov in pri drugih realnih problemih. Matematični pojmi bodo predstavljeni predvsem z uporabnega zornega kota, poudarek bo na konkretnih primerih in

#### **Objectives and competences:**

The aim of this course is to introduce in an informal and intuitive way the basic concepts of algebraic topology which are used in algorithms for analysis of big, possibly higher dimensional data sets, for reconstruction of objects and configuration spaces of robots and mechanical systems and in other practical applications. Mathematical concept will be presented from the point of view of applications, special attention will be given to specific examples and

računalniških algoritmih.

algorithms.

**Predvideni študijski rezultati:**

Znanje in razumevanje:

Znanje s področja matematike, ožje topologije in algebraične topologije, ki je uporabno pri analizi podatkov.

Razumevanje pomena matematičnega modela in njegovih lastnosti pri reševanju konkretnih problemov.

Uporaba:

Poznavanje osnovnih računskih prijemov in algoritmov za iskanje rešitev. Implementacija algoritmov in uporaba na konkremtnem primeru.

Refleksija:

Kritična analiza dobljenih rezultatov.

Aplikacija matematičnih rešitev na razumevanje konkretnega problema.

Prenosljive spretnosti - niso vezane le na en

predmet:

Razumevanje pomena izbire računskega modela glede na dane podatke in željene rezultate.

**Intended learning outcomes:**

Knowledge and understanding:

Understanding the necessary mathematics, specifically algebraic topology, which can be used in data analysis.

Understandind the role of the mathematical model and its properties in solving specific applied problems.

Application:

Knowledge of the basic computational approaches and algorithms for finding solutions. Implementation of the algorithms and application to specific examples.

Reflection:

A critical analysis of the results.

Application of the mathematical solution to understanding the original problem.

Transferable skills:

Understanding the significance of the choice of the computational model with respect to the given data and expected results.

**Metode poučevanja in učenja:**

**Learning and teaching methods:**

Predavanja s podporo avdio-vizualne opreme, predstavitev teoretičnih pojmov in prikaz pomena na konkretnih primerih, laboratorijske vaje v računalniški učilnici z ustrezno programsko opremo. Delo posamezno in v skupinah. Velik poudarek na praktičnem delu in na skupinskem reševanju praktičnih problemov.	Combined lecturing with simultaneous use of the blackboard and computer projection explaining the theoretical concepts and specific meaning in specific cases. Lab work in computer-equipped lecture rooms. Individual and work in team. Emphasis on practical problem solving and group work.
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Načini ocenjevanja:	Delež (v %) / Weight (in %)	Assessment:
Način (pisni izpit, ustno izpraševanje, naloge, projekt):		Type (examination, oral, coursework, project):
Sprotno preverjanje (domače naloge, kolokviji in projektno delo)		Continuing (homework, midterm exams, project work)
Končno preverjanje (pisni in ustni izpit)	50%	Final: (written and oral exam)
Ocene: 6-10 pozitivno, 1-5 negativno (v skladu s Statutom UL)	50%	Grading: 6-10 pass, 1-5 fail.

#### Reference nosilca / Lecturer's references:

- JURČIČ-ZLOBEC, Borut, MRAMOR KOSTA, Neža. Geometric constructions on cycles. Rocky Mountain journal of mathematics, ISSN 0035-7596, 2004, vol. 34, no. 4, str. 1565-1585. [COBISS.SI-ID 13268057]
- KING, Henry C., KNUDSON, Kevin, MRAMOR KOSTA, Neža. Generating discrete Morse functions from point data. Experimental mathematics, ISSN 1058-6458, 2005, vol. 14, no. 4, str. 435-444. [COBISS.SI-ID 13872985]
- JAWOROWSKI, Jan, MRAMOR KOSTA, Neža. The degree of maps of free G-manifolds. Journal of fixed point theory and its applications, ISSN 1661-7738, 2007, vol. 2, no. 2, str. 209-213. [COBISS.SI-ID 14569305]
- JERŠE, Gregor, MRAMOR KOSTA, Neža. Ascending and descending regions of a discrete Morse function. Computational geometry, ISSN 0925-7721. [Print ed.], 2009, vol. 42, iss. 6-7, str. 639-651. [COBISS.SI-ID 14994265]
- AYALA, Rafael, VILCHES, Jose Antonio, JERŠE, Gregor, MRAMOR KOSTA, Neža. Discrete gradient

fields on infinite complexes. Discrete and continuous dynamical systems, ISSN 1078-0947, 2011,  
vol. 30, no. 3, str. 623-639. [COBISS.SI-ID 15865945]